On restoring the parameters of the wave equation using a known boundary condition

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We considered the problem of obtaining the unknown functions $f(x)$ and $g(x)$ from the distribution of the derivative of the coordinate on an arbitrary segment in the Cauchy problem for the hyperbolic equation:

$$\begin{cases} \dfrac{\partial^2 u(x,t)}{\partial t^2} = \dfrac{\partial^2 u(x,t)}{\partial x^2}, x \in (0;1), t \in (0;1), \\ u(0,t) = 0, t \in [0;1], \quad u(1,t) = 0, t \in [0;1], \quad \dfrac{\partial u(1,t)}{=} h(t), t \in [0;1], \\ u(x,0) = f(x), x \in [0;1], \quad \dfrac{\partial u(0,t)}{\partial t} = g(x), x \in [0;1], \end{cases}$$

where the function $h(t) \in H^3[0;1]$, and you need to find the unknown functions $f(x) \in H_0^4[0;1]$, $g(x) \in H_0^3[0;1]$.

The solution to the inverse problem is correct and can be found quite simply: $f(x) = \sum\limits_{n=1}^{\infty} \dfrac{a_n}{n} \sin(\pi n x)$, $displaystyle g(x) = \sum\limits_{n=1}^{\infty} b_n \sin(\pi n x)$, where a_n, b_n are the corresponding expansion coefficients of the function $h(t)$ in Fourier series:

$$a_n = \frac{2}{T} \int\limits_0^T h(t) \cos\left(\frac{\pi n t}{T}\right), \quad b_n = \frac{2}{T} \int\limits_0^T \sin\left(\frac{\pi n t}{T}\right),$$

where T is the period of the function $h(t)$, $T \leq 1$.

This problem is used to reconstruct the pattern of electromechanical vibrations of long polymer oxyhydrate molecules from the measured electric current.