Statement, solution and study of the direct problem for the wave equation on a finite time interval

Марков Борис Анатольевич
Южно-Уральский государственный университет (Челябинск), Россия
e-mail: smpx1969@mail.ru

Tanana Vitalii Pavlovich
Южно-Уральский государственный университет (Челябинск), Россия
e-mail: tananavp@susu.ru

Сухарев Юрий Иванович
Южно-Уральский государственный университет (Челябинск), Россия

The Dirichlet problem is given for the time of oscillations of a linear fragment of a colloidal substance on a unit time interval:
\begin{eqnarray}\label{eqn1-1}
\left\{
\begin{array}{lll}
\displaystyle\frac{\partial^2 u(x, t)}{\partial t^2}=\frac{\partial^2u(x, t)}{\partial x^2}, \quad x\in (0; pi), \quad t \in (0; 1),\\
\displaystyle u(0, t)=0, \quad t\in [0; 1], \quad u(1, t)=0, t\in [0; 1];\\
\displaystyle u(x, 0) = f(x), \quad x\in [0; \pi], \quad u(x, T)=g(x), \quad x\in [0; \pi],\\
f(0)=f(\pi)=f”(0)=f”(\pi)=g(0)=g”(0)=g”(\pi)=g(\pi)=0,
\end{array}\right.
\end{eqnarray}
where the functions $f(x)\in H^{10}[0; 1],$ $g(x)\in H^{10} [0; 1],$ $T>0$ — time interval of measurements, $u(x, t)$ – deviation of the linear fragment rod from the equilibrium position. The solution to the problem (\ref{eqn1-1}) will be considered a classical solution, which is unique and stable according to the initial data.

The complexity of the problem (\ref{eqn1-1}) is that its solution for certain lengths of the time interval is not unique \cite{Ivanov} or may not exist at all (we choose the case corresponding to the value $\alpha=1/\ pi$ in \cite{Ivanov} notation).

A solution to the problem (\ref{eqn1-1}) exists and is unique under these conditions. Moreover, it is not equivalent to the Cauchy problem in time, since for the existence of a classical solution it is sufficient that $f(x)\in H^4[0;1],$ $h(x)\in H^3[0; 1].$ The results given in the original statement (it is cited in \cite{Ivanov}) are doubtful.