Solving the inverse problem of designing cloaking devices using machine learning

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Theory and numerical methods for solving inverse and ill-posed problems
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The statement of the design problem of the magnetic cloaking (or shielding) device includes three components: the main domain D, the given magnetic field H_a created by the external sources and the cloaking (or shielding) shell having the spherical form filled with special inhomogeneous medium.

Forward problem

The direct problem of finding the total field $\Phi = \Phi_a + \Phi_s$ reduces to finding all M + 2 fields Φ_m in domains Ω_m, $m = 0$; M + 1 by solving the following magnetic conjugation problem:

$$\Delta\Phi_0 = 0 \text{ in } \Omega_0, \;\; \Delta\Phi_{M+1} = 0 \text{ in } \Omega_{M+1},$$

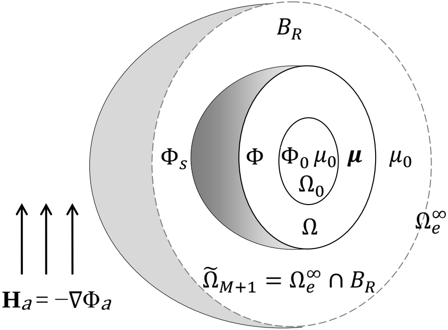
$$\mathrm{div}\,(\mu_m \nabla \Phi_m) = 0 \text{ in } \Omega_m, \quad m = \overline{1, M}, \tag{1}$$

$$\Phi_m = \Phi_{m+1}, \;\; \mu_{rm}\frac{\partial \Phi_m}{\partial r} = \mu_{r(m+1)}\frac{\partial \Phi_{m+1}}{\partial r} \text{ on } r = R_m, \;\; m = \overline{0, M}, \tag{2}$$

$$\Phi_0(\mathbf{x}) = O(1) \text{ as } r = |\mathbf{x}| \to 0, \quad \Phi_{M+1}(\mathbf{x}) \to \Phi_a(\mathbf{x}) \text{ as } r \to \infty, \tag{3}$$

Here $\mu_{r0} = \mu_{r(M+1)} = \mu_0$ is constant magnetic permeability of homogeneous isotropic medium filling domains Ω_0 and Ω_{M+1}.
Field Φ_m is the restriction $\Phi|_{\Omega_m}$ of the total field $\Phi = \Phi_a + \Phi_s$ to the subdomain Ω_m, $m = \overline{0, M+1}$. The solution exists and is unique[1]

[1] G.V. Alekseev and Y.E. Spivak // Diff. Eq. **54**. 2018.



Schematic representation of an external applied magnetic field and a ball B_R of radius R containing a spherical magnetic shell $\Omega = \{\mathbf{x} \in \mathbb{R}^3 : a < r = |\mathbf{x}| < b\}$.

Statement of the inverse problem

We define cost functionals:

$$J_i(\mathbf{m}) = \frac{\|\nabla\Phi_0[\mathbf{m}]\|_{L^2(\Omega_0)}}{\|\nabla\Phi_a\|_{L^2(\Omega_0)}}, \quad J_e(\mathbf{m}) = \frac{\|\Phi_{M+1}[\mathbf{m}] - \Phi_a\|_{L^2(\Omega_{M+1})}}{\|\Phi_a\|_{L^2(\Omega_{M+1})}}.$$

The inverse shielding and cloaking problems are

$$J_i(\mathbf{m}) \to \inf, \quad \mathbf{m} \in K, \tag{4}$$

$$J(\mathbf{m}) = (1/2)[J_i(\mathbf{m}) + J_e(\mathbf{m})] \to \inf, \quad \mathbf{m} \in K. \tag{5}$$

K is called the control set:

$$K = \{\mathbf{m} = (\mu_1, \mu_2, ..., \mu_M) \in \mathbb{R}^3 : 0 < \mu_{min} \leq (\mu_m) \leq \mu_{max}\}. \tag{6}$$

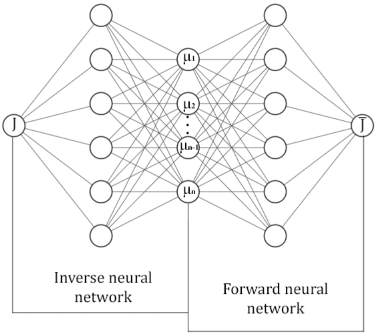
The value $J_i(\mathbf{m})$ is associated with shielding performance of the cloak.

The value $J(\mathbf{m})$ corresponds to the cloaking performance of the device.

Solving the inverse problem of designing cloaking devices using machine learning

It is possible to solve the inverse problem of designing cloaking devices using inverse neural network. This network will generate magnetic permeabilities $\mathbf{m} = \{\mu_1, .., \mu_M\}$ of shell layers corresponding to a given quality functional $J(\mathbf{m})$ value.

To train the inverse network,a pre-trained forward network is needed, as it is shown at the figure below.



We are using the following metrics to assess the neural network prediction quality: Coefficient of determination, R^2:

$$\mathrm{R}^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - x_i)}{\sum_{i=1}^{n} (y_i - \overline{y})}.$$

Mean absolute error, MAE:

$$\mathrm{SMAPE} = \frac{100}{n} \sum_{i=1}^{n} \frac{|y_i - x_i|}{|y_i| + |x_i|}.$$

where y_i is the predicted value, x_i is the numerical simulation value and $\overline{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$.

Machine learning without using a priori information

When using machine learning without a priori information, it is necessary to determine the input parameters and the limits of their variation. Let's consider the case of 6-layers cloaking device.

We are using magnetic permeabilities for each layer of the cloaking device, $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6 \in [0.008, 70]$ as input parameters. A dataset total of 20 000 solutions of forward problems was generated. Forward neural network was trained. Inverse network was trained using forward network. The results are as follows.

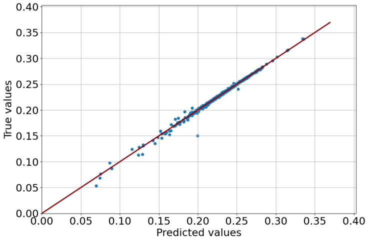
Forward network:

$$\mathrm{R}^2 = 0.994, \ \mathrm{SMAPE} = 0.15\%.$$

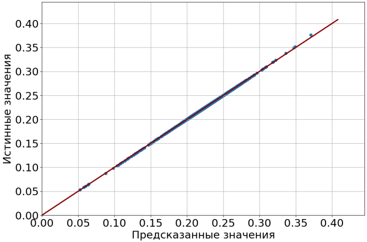
Inverse network:

$$\mathrm{R}^2 = 0.999, \ \mathrm{SMAPE} = 0.04\%.$$

Correspondence plot of predicted and true values for the forward and inverse models are presented at the figure below.



Forward network



Inverse network

Machine learning without using a priori information

Optimization results are as follows:

	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	J_i	J_e	J
NN	18.45	36.57	48.34	$1.7 \cdot 10^{-2}$	22.24	14.54	$4 \cdot 10^{-2}$	$1.08 \cdot 10^{-1}$	$7.36 \cdot 10^{-2}$
PSM	$4.5 \cdot 10^{-3}$	70	$4.5 \cdot 10^{-3}$	70	$4.5 \cdot 10^{-3}$	13.23	$6.8 \cdot 10^{-5}$	0	$3.4 \cdot 10^{-5}$

It is shown that without using a priori information machine learning methods worse produce worse results than particle swarm method.

Machine learning using a priori information

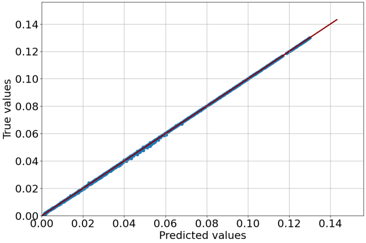
We use the following information as a priori:
1) The alternating design gives the best results.
2) The more layers – the better the results.
We are using magnetic permeabilities for alternating design of layers of the cloaking device, $\mu_{min} \in [0.002, 0.01], \mu_{max} \in [30, 70], \mu_M \in [0.002, 70]$ as well as number of layers M an as input parameters.
A dataset total of 20 000 solutions of forward problems was generated. Forward and inverse networks were trained. The results are as follows.
Forward network:

$$\mathrm{R}^2 = 0.999,\ \mathrm{SMAPE} = 0.18\%.$$

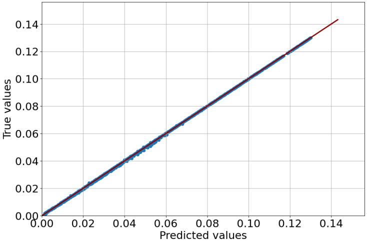
Inverse network:

$$\mathrm{R}^2 = 0.999,\ \mathrm{SMAPE} = 0.36\%.$$

Correspondence plot of predicted and true values for the forward model and inverse model are presented at the figure below.



Forward network



Inverse network

Machine learning using a priori information

Optimization results are as follows:

	μ_{min}	μ_{max}	μ_M	M	J_i	J_e	J
NN	$2 \cdot 10^{-3}$	70	14.84	12	$1.93 \cdot 10^{-12}$	$3.8 \cdot 10^{-3}$	$1.87 \cdot 10^{-3}$
PSM	$4.5 \cdot 10^{-3}$	70	28.16	16	$1.68 \cdot 10^{-7}$	0	$8.34 \cdot 10^{-8}$

It is shown that machine learning might produce better results for shielding problem, but overall results are worse than particle swarm method results.

Thank you for your attention!