ON INVERSION OF THE NORMAL RADON TRANSFORM, ACTING ON 3D TENSOR FIELDS, USING SV-DECOMPOSITION

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The longitudinal ray transform is a classical integral geometry operator acting on vector and symmetric m-tensor fields [1]. In $\mathbb{R}^3$, only the solenoidal part of a vector or symmetric m-tensor field ($m \geqslant 2$) can be reconstructed from the longitudinal ray transform. To restore the potential part, we must have other data. One of operators that makes it possible to reconstruct the potential part of a tensor field is the normal Radon transform. In [2] the problem of inverting the Radon transform of a symmetric 2-tensor field and, in particular, the normal Radon transform was investigated.

We consider the integral geometry problem of finding a potential part of 3D symmetric m-tensor field provided that the normal Radon transform is known. In other words, we look for the inverses of normal Radon transform applied to a tensor field in $\mathbb{R}^3$. That is, it is required to solve the operator equation $Af = g$, where A is a linear bounded (integral in our case) operator, f is an unknown symmetric m-tensor field and g is a known right side, which is an initial data of the problem.

In the singular value decomposition (SVD) method for inverting operators, the operator A is represented as a series of singular values and basis elements in the space of images. Then the (pseudo)inverse operator is represented, using singular functions, as a series of similar structure with preimages of the basis elements and the inverse of the singular values. The result of the work [3] became the basis for constructing singular value decompositions of other integral operators. Namely, SVD of the normal Radon transform acting on vector [4] and symmetric 2-tensor fields [5] in $\mathbb{R}^3$, and also the result of current report were obtained.

In the report, properties of the normal Radon transform acting on 3D tensor fields are investigated and a singular value decomposition of the operator is obtained. Based on the decompositions the polynomial expressions for the (pseudo)inverse and adjoint operators are obtained.

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