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Regularization and uniqueness of solutions of systems Fredholm-Stieltjes linear integral equations of the first kind with two variables

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This stientific reports is devoted to the study of the regularization and uniqueness of solutions of systems Fredholm-Stieltjes linear integral equations of the first kind with two independent variables in which the operator generated by the kernel, is not compact operator.

Various issues concerning of integral equations of the first kind were studied in [1-7]. Some practical and theoretical investigations were made in paper [1] for nonclassical Volterra integral equations of the first kind. In [2] for system of Volterra integral equations of the first kind were constructed the Volterra regularized operators. In [3] the paper a theorem is proved the uniqueness of the solution of Fredholm-Stielties integral equations of the rst kind with two independent variables. In [4] for the systems of Volterra integral equations of the first kind with two independent variables were investigated the problems of regularization and uniqueness. More specifically, fundamental results for Fredholm integral equations of the first kind were obtained in [6], where regularizing operators in the sense of M.M.Lavrent'ev vere constructed for solutions of linear Fredholm integral equations of the first kind. In this work for the investigation of the integral equation we it is based on the notion of the derivative of function with respect to the strictly increasing function [7]. Apparently the first notion of the derivative, with respect to the strictly increasing function, was introduced in [7]. In this paper, while analizing the following integral equation.

$$\int_a^b K(t,x,y)u(t,y)d\varphi(y) + \int_{t_0}^T Q(t,x,s)u(s,x)d\psi(s)+$$

$$\int_{t_0}^t \int_a^x C(t,x,s,y)u(s,y)d\varphi(y)d\psi(s) =$$

$$= f(t,x),\ (t,x) \in G = \left\{(t,x) \in R^2 : t_0 \le t \le T,\ a \le x \le b\right\}, \tag{1}$$

where

$$K(t,x,y) = \left\{ \begin{array}{l} A(t,x,y),\ t_0 \le t \le T,\ a \le y \le x \le b; \\ \\ B(t,x,y),\ t_0 \le t \le T,\ a \le x \le y \le b, \end{array} \right\} \tag{2}$$

$$Q(t,x,s)=\left\{\begin{array}{l} M(t,x,s),\ t_0\le s\le t\le T,\ a\le x\le b, \\ \\ N(t,x,s),\ t_0\le t\le s\le T,\ a\le x\le b, \end{array}\right\} \tag{3}$$

$A(t,x,y),B(t,x,y),M(t,x,s),N(t,x,s),C(t,x,s,y)$ - are given continuous functions, respectively on the domains

$$G_1=\{(t,x,y):t_0\le t\le T,\ a\le y\le x\le b\};$$
$$G_2=\{(t,x,y):t_0\le t\le T,\ a\le x\le y\le b\};$$
$$G_3=\{(t,x,s):t_0\le s\le t\le T,\ a\le x\le b\};$$
$$G_4=\{(t,x,s):t_0\le t\le s\le T,\ a\le x\le b\};$$
$$G_5=\{(t,x,s,y):t_0\le t\le s\le T,\ a\le x\le b\}.$$

$u(t,x)$ and $f(t,x)$ are the unknown and given functions respectively, $(t,x)\in G$. $\varphi(x)$ is the increasing continuous function in $[a,b],\psi(t)$ is the increasing continuous function in $[t_0,T]$.

Using $A(t,x,y),B(t,x,y),M(t,x,s)$ and $N(t,x,s)$ we define the following functions

$$\left\{\begin{array}{l} P(t,x,y)=A(t,x,y)+B(t,y,x),(t,x,y)\in G_1; \\ \\ H(t,x,s)=M(t,x,s)+N(s,x,t),(t,x,s)\in G_3 \end{array}\right\} \tag{4}$$

References:

1. Apartsyn, A.S. (2003). Nonclassical linear volterra equations of the first kind, Utrecht-Boston,The Netherlands:VSP, 168p.
2. Asanov, A. (1998). Regularization, uniqueness and existence of solutions of Volterra equations of the first kind, Utrecht-, The Netherlands: VSP, 282p.
3. Asanov, A. Kadenova, Z.A.(2019). Uniqueness of solutions for one class of Fredholm - Stieltjes equations of the first kind with tho variables, Contemporary problems of mathematics and mechanics. Proceedings of the conference dedicated to the 80th anniversary of academician V. A. Sadovnichy. - Moscow: MAKS Press, 2019, pp.177-179.
4. Imanaliev M.I. and Asanov A. On solutions of systems of nonlinear two-dimensional Volterra integral equations of the first kind // Dokl. Akad. Nauk. SSSR, 317(1), 330-333. (1991).
5. Imanaliev, M.I. Asanov, A.Kadenova, Z.A.(2014). A Class of Linear Integral Equations of the First Kind with Two Independent Variables, Doklady Mathematics, vol.89, no.1, pp.98-102.
6.Lavrent'ev, M.M. (1959). The integral equations of the first kind, Doklady Akademii Nauk SSSR, vol.127, no.1,pp.31-33(in Russian).
7. Asanov, A. Kadenova, Z.A.,Orozmamatova J.(2020).Uniqueness and stability of solutions for certain linear equations of the third kind with two variables,Herald of institute mathematics of the National Academy of Sciences of the Kyrgyz Republic, no.2, pp.63-73.
8. Asanov A. The denivative of a function by means of a increasing function. Fen Bilimleri Dergisi, Kyrgyz-Turkish Manas University, 1. 18-64 (2001).