Solitons and spin waves in spiral structures

Kiselev V.V., Rascovalov A.A.,
(Institute of Metal Physics)

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Introduction

At the last few years many optically transparent magnets with a spiral structure have been synthesized.

The Landau-Lifshitz equations for quasi-one-dimensional spiral with some restrictions can be reduced to sine-Gordon equation. Sine-Gordon equation is well-known, but usually this equation is solved against the homogeneous background.

In our case, spiral structure represents essentially nonlinear inhomogeneous ground state. So far, nonlinear dynamics of solitons and spin waves in such structures is not investigated. It is known only most simple soliton solution, which was found by Borisov and Ovchinnikov at 2009.

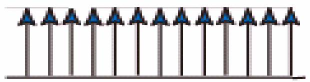
Sine-Gordon equaiton. The choice of spiral ground state

In dimensionless variables: $\partial_t^2\Phi - \partial_z^2\Phi + \sin\Phi = 0$,

$$\boxed{w = \frac{\tilde{w}}{M_0 H} = \frac{1}{2}\left[(\partial_z\Phi)^2 + (\partial_t\Phi)^2\right] + q\partial_z\Phi + (1-\cos\Phi)} \, ,$$

$\boxed{q < 4/\pi}$

$\Phi = 0 \quad (\mathrm{mod}\ 2\pi)$



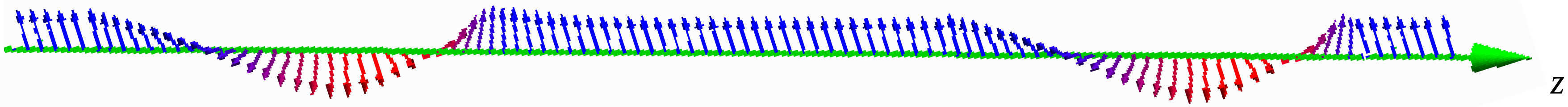
$\mathbf{M} = M_0(1,0,0)$

$\boxed{q > 4/\pi}$

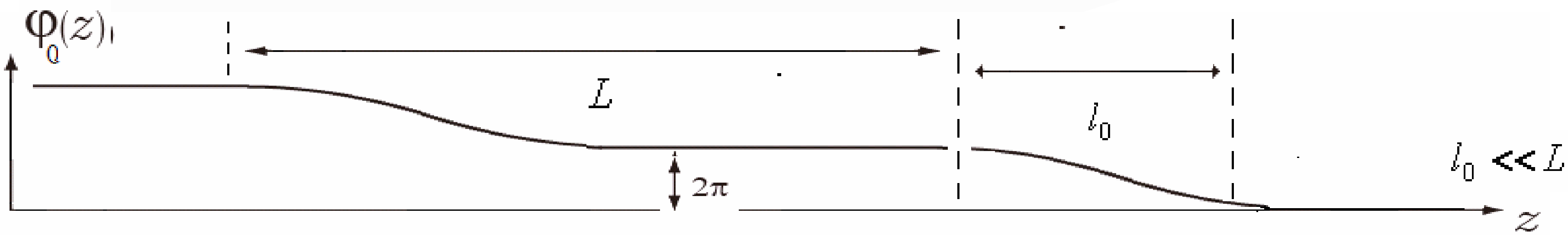
$\mathbf{M} \approx M_0(\cos\Phi, \sin\Phi, 0)$

$\boxed{\Phi \equiv \varphi_0(\chi) = \pi - 2\mathrm{am}(\chi, k)}$

$(\chi = z/k)$



spiral (stripe domain) structure with walls of one and the same chirality



kink lattice

Difficulties, inherent to Riemann's problem for helicoidal structure

- ✓ **The Riemann's problem is formulated on the two-sheets Riemann's surface, related with spiral structure.**

- ✓ **Because of the periodic background, the functions of Riemann's problem appear to be "Bloch functions": they acquire additional multipliers every time, when coordinate is shifted on period.**

- ✓ **The formation of solitons is accompanied by the macroscopic translations of helicoidal structure. These translations explicitly appear in boundary conditions.**

Advantages of “dressing” technique

- ✓ **Final formulas do not contain transcendental relations and multidimensional theta-functions (inherent to finite-gap-integration technique), and are expressed in terms of well-known <u>Jacobi elliptic functions</u>.**

- ✓ **We do not use integral transformations: technique of “dressing” is <u>local</u>.**

- ✓ **This technique describes not only solitons, but also <u>spin waves</u> <u>at arbitrary initial distributions of magnetization</u> in spiral structure.**

- ✓ **We can find spectral expressions of integrals of motion for collective excitations in helicoidal structure.**

“Dressing” technique for helicoidal structure: 1. Auxiliary linear system

Equation $\partial_t^2\Phi - \partial_z^2\Phi + \sin\Phi = 0$ is equivalent to compactibility condition of system

$$\partial_t \Psi = \frac{\mathrm{i}}{2}\left[\frac{\partial_z \Phi}{2}\sigma_3 + \sigma_1\omega_1 \cos\frac{\Phi}{2} + \sigma_2\omega_2 \sin\frac{\Phi}{2}\right]\Psi \equiv V\Psi,$$

$$\partial_z \Psi = \frac{\mathrm{i}}{2}\left[\frac{\partial_t \Phi}{2}\sigma_3 + \sigma_1\omega_2 \cos\frac{\Phi}{2} + \sigma_2\omega_1 \sin\frac{\Phi}{2}\right]\Psi \equiv U\Psi.$$

(L.A. Tachtadgan, L.D. Faddeev)

In parametrization $\omega_1 = \mathrm{cn}(u,k),\ \omega_2 = \mathrm{isn}(u,k)$:

$$\varphi_0(\chi+\delta) = \pi - 2\mathrm{am}(\chi+\delta,k) \quad (\chi = z/k) \Rightarrow \Psi_\delta(\chi,t,u) = M(u,\chi+\delta)\exp\left(A(u,\chi,t)\sigma_3 + \frac{\eta_1\delta}{K}u\sigma_3\right)$$

where
$$M(u,\chi) = \frac{m(\chi)}{\sigma(u)}\begin{pmatrix} \sigma(\chi+u) & \mathrm{i}\sigma(\chi+2\mathrm{i}K'-u)e^{-\eta_3(\chi+\mathrm{i}K'-u)} \\ -\mathrm{i}\sigma(\chi+2\mathrm{i}K'+u)e^{-\eta_3(\chi+\mathrm{i}K'+u)} & \sigma(\chi-u)\end{pmatrix}\exp\left[-\frac{\eta_1 u}{k}\chi\sigma_3\right];$$

$$m(\chi) = \frac{1}{\sqrt{2}}\left|\frac{\sigma(\mathrm{i}K')}{\sigma(\chi+\mathrm{i}K')}\right|; \qquad \det M = \frac{\mathrm{dn}u}{1-\mathrm{dn}u}, \qquad A(u,\chi,t) = \mathrm{i}p(u)\chi + \frac{\mathrm{i}t}{2K}\mathrm{dn}(u,k)$$

$p(u) = \frac{\mathrm{i}}{2}Z(u)$ -“Bloch quasi- -momentum”

$$\Psi_\delta(\chi \pm 4K,t,u) = \Psi_\delta(\chi,t,u)\exp(\pm 4\mathrm{i}Kp(u)\sigma_3)$$

2. Solitons lead to the macroscopic shift Δ in boundary conditions

number of kinks

odd $2M+1$ $(\sigma = \pm 1)$

even $2M$

$$\Phi(z,t) \to \varphi_1^{(0)}(\chi) \equiv 2\pi\sigma + \varphi_0(\chi + \Delta) \quad \text{at} \quad z \to -\infty$$

$$\Phi(z,t) \to \varphi_2^{(0)}(\chi) \equiv \varphi_0(\chi) \quad \text{at} \quad z \to \infty$$

$$\Phi(z,t) \to \varphi_1^{(0)}(\chi) \equiv \varphi_0(\chi + \Delta) \quad \text{at} \quad z \to -\infty$$

$$\Phi(z,t) \to \varphi_2^{(0)}(\chi) \equiv \varphi_0(\chi) \quad \text{at} \quad z \to \infty$$

(*M* pairs of kinks have mutually opposite topological charges)

+ arbitrary number of breathers (bound state of two kinks <u>with opposite topological charges</u>) + spin waves

$$\varphi_1^{(0)}(\chi) \Rightarrow \Psi_1^{(0)} = \sigma_3 \Psi_\Delta(z,t,u)\sigma_3$$

$$\varphi_2^{(0)}(\chi) \Rightarrow \Psi_2^{(0)} = \Psi_0(z,t,u)$$

$$\varphi_1^{(0)}(\chi) \Rightarrow \Psi_1^{(0)} = \Psi_\Delta(z,t,u)$$

$$\varphi_2^{(0)}(\chi) \Rightarrow \Psi_2^{(0)} = \Psi_0(z,t,u)$$

Yost functions:

$$\boxed{\begin{aligned} &\Psi_1(z,t,u) \to \Psi_1^{(0)} \quad \text{at} \quad z \to -\infty \\ &\Psi_2(z,t,u) \to \Psi_2^{(0)} \quad \text{at} \quad z \to \infty \end{aligned}}$$

3. Transition matrix and continuous spectrum

On contour $\gamma = \{u : \operatorname{Im} p(u) = 0\}$ Yost functions are related by the transition matrix:

$$\boxed{\Psi_1(u) = \Psi_2(u) T(u)};$$

$$\boxed{T(u) = \begin{pmatrix} a(u) & -b^*(-u^*) \\ b(u) & a^*(-u^*) \end{pmatrix}}$$

$$a(u)a^*(-u^*) + b(u)b^*(-u^*) = 1\,;$$

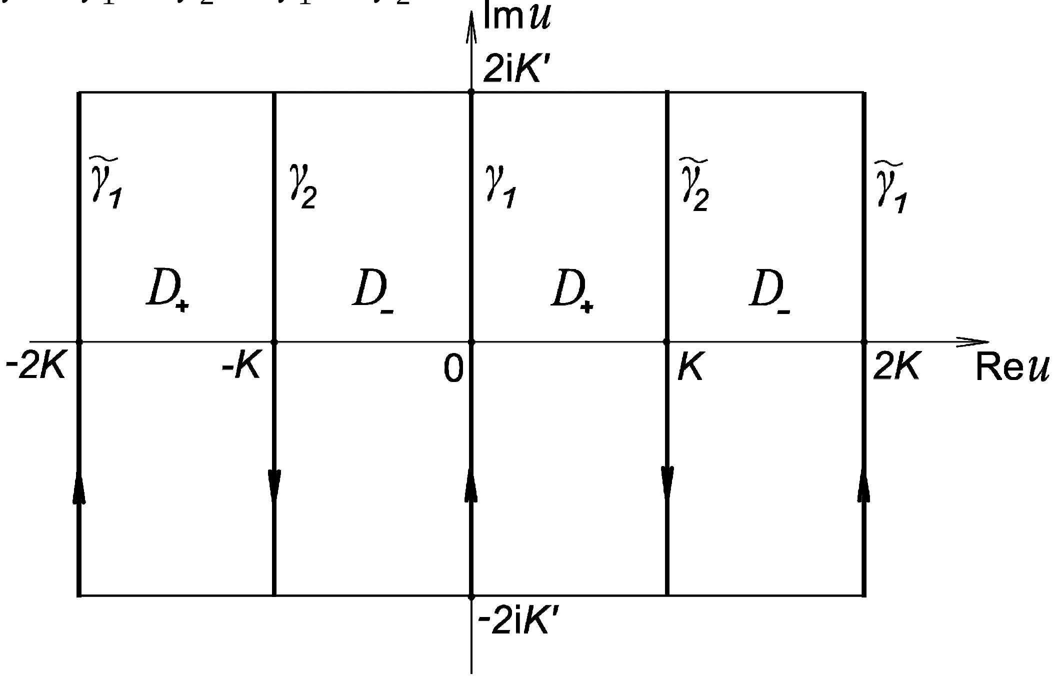
$$a(u \pm 2K) = a(u)e^{\pm 2\eta_1 \Delta}\,, \quad \theta = \pm 1,$$

$$a^*[(u \pm 2\mathrm{i}K')^*] = -\theta a(u) e^{\pm \eta_3 \Delta}\,;$$

$$b(u \pm 2K) = -b(u)e^{\pm 2\eta_1 \Delta}\,,$$

$$b^*[(u \pm 2\mathrm{i}K')^*] = -\theta b(u) e^{\pm \eta_3 \Delta}\,.$$

$$\gamma = \gamma_1 \cup \gamma_2 \cup \tilde{\gamma}_1 \cup \tilde{\gamma}_2$$



To apply “dressing” technique, it is useful to recombine the columns of the Yost functions into new functions $\Psi_-(u) = \left(\Psi_1^{(1)}(u), \Psi_2^{(2)}(u)\right)$ $\Psi_+(u) = \left(\Psi_2^{(1)}(u), \Psi_1^{(2)}(u)\right)$, which are analytical in the regions D_- and D_+ accordingly.

4. The formulation of Riemann's problem

To find two functions $\Psi_+(u)$ and $\Psi_-(u)$, which are analytical in regions D_+ and D_- , whereas they satisfy following conjugation condition:

$$\boxed{\Psi_-(u)=\frac{\Psi_+(u)}{a^*(-u^*)}\begin{pmatrix}1 & b^*(-u^*)\\ b(u) & 1\end{pmatrix}, \qquad u\in\gamma.}$$

reductions

$$\Psi_+(u\pm 2K)=\sigma_3\Psi_+(u)\sigma_3\begin{pmatrix}1 & 0\\ 0 & e^{\mp 2\eta_1\Delta}\end{pmatrix},\quad \Psi_+^*[(u\pm 2\mathrm{i}K')^*]=q(u)\sigma_1\Psi_+(u)\sigma_3\begin{pmatrix}1 & 0\\ 0 & e^{\mp\eta_3\Delta}\end{pmatrix},$$

$$\Psi_-(u\pm 2K)=\sigma_3\Psi_-(u)\sigma_3\begin{pmatrix}e^{\pm 2\eta_1\Delta} & 0\\ 0 & 1\end{pmatrix};\quad \Psi_-^*[(u\pm 2\mathrm{i}K')^*]=-\theta q(u)\sigma_1\Psi_-(u)\sigma_3\begin{pmatrix}e^{\pm\eta_3\Delta} & 0\\ 0 & 1\end{pmatrix};$$

and restriction $\boxed{\Psi_+(u)=-\sigma_2\Psi_-^*(-u^*)\sigma_2, \qquad u\in\gamma.}$

$$q(u)=\frac{k\,\mathrm{sn}u}{1+\mathrm{dn}u}$$

$\Psi_+(u),\ \Psi_-(u)$ are quasi-periodic

5. Solution Φ(z,t) of sine-Gordon is found from expansion:

$$\Psi_{-}(u = \mathrm{i}K' + \varepsilon) = \frac{\mathrm{i}}{\sqrt{2}} \exp\left[\frac{\mathrm{i}}{4}(\Phi + \pi)\sigma_3\right](1 + \mathrm{i}\sigma_1) \times$$

$$\times \exp\left[-\frac{(z-t)}{2k\varepsilon}\sigma_3\right] \mathrm{diag}\left(\mathrm{i}\sigma \exp\left[\frac{\eta_3 \Delta}{2}\right], \quad 1\right) + O(\varepsilon),$$

$$\sigma = \pm 1.$$

6. Solitons

$b(u), b^*(-u^*) \equiv 0$ and $a(u), a^*(-u^*)$ have zeroes in their analyticity regions

Zeroes of $a(u)$ are divided into two groups:

$a)\quad u = v_p + \mathrm{i}\varepsilon_p K' \equiv v_p, \quad -K < v_p < 0, \quad \varepsilon_p = \pm 1; \quad p = 1,2\ldots m;$ → kinks

$b)\quad u = \mu_s, \quad \mu_s^* - 2\mathrm{i}K', \quad -K < \mathrm{Re}\mu_s < 0; \quad s = 1,2\ldots n.$ → breathers

$$a(u) = \prod_p \varepsilon_p \frac{\sigma(u - v_p - \mathrm{i}\varepsilon_p K')}{\sigma(u + v_p - \mathrm{i}\varepsilon_p K')} e^{-\eta_3 \varepsilon_p v_p} \prod_s \frac{\sigma(u - \mu_s)\sigma(u - \mu_s^* + 2\mathrm{i}K')}{\sigma(u + \mu_s^*)\sigma(u + \mu_s + 2\mathrm{i}K')} e^{\eta_3(\mu_s + \mu_s^*)} \qquad [2K, 4\mathrm{i}K']$$

Zeroes are related with the <u>macroscopic shift</u> Δ by formula:

$$\sum_p v_p + \sum_s (\mu_s + \mu_s^*) = -\Delta/2, \qquad \mathrm{mod}(2K).$$

Pecularities of “dressing” for solitons

1) The function $\Psi_-(u)$ cannot be normalized on unit matrix. Hence, we must use asymptotic values of $\Psi_-(u)$ at $z \to \pm\infty$.

2) As the functions of Riemann’s problem are not doubly periodic on *u* , instead of Weierstrass $\zeta(u)$ - functions, we use:

$$\kappa(\mu) = \frac{\sigma(\mu+\mu^*)\sigma(\mu-\mu^*-2\mathrm{i}K')}{\sigma(\mu+\mathrm{i}K')\sigma(\mu-\mathrm{i}K')}\exp(\eta_3\mu)\ , \qquad g_1(u) = \frac{\sigma(u-\mu)}{\sigma(u+\mu^*)}\exp\left[-\frac{\eta_3}{2}(\mu+\mu^*)\right]$$

$$g_3(u) = \mathrm{i}\frac{\sigma(u+\mathrm{i}K')\sigma(u-\mathrm{i}K')}{\sigma(u+\mu^*)\sigma(u+\mu-2\mathrm{i}K')}\exp\left[-\eta_3\left(u+\frac{\mu+\mu^*}{2}\right)\right], \qquad g_2(u) = \frac{\sigma(\mu-\mu^*-2\mathrm{i}K')}{\sigma(u+\mu-2\mathrm{i}K')}\exp\left[-\frac{\eta_3}{2}(\mu+\mu^*)\right]$$

$$g_{1,2}(u\pm 2K) = g_{1,2}(u)e^{\mp 2\eta_1(\mu+\mu^*)},\ g_3(u\pm 2K) = -g_3(u)e^{\mp 2\eta_1(\mu+\mu^*)};$$

$$g_{1,2}^*[(u\pm 2\mathrm{i}K')^*] = g_{2,1}(u)e^{\mp\eta_3(\mu+\mu^*)},\ g_3^*[(u\pm 2\mathrm{i}K')^*] = g_3(u)e^{\mp\eta_3(\mu+\mu^*)};$$

3) $\Psi_-(u)$ is not expressed in terms of projector matrices.

4) We have constructed multisoliton solutions by recurrent way. Multisoliton matrices $\Psi_\pm(u)$ are factorized and represent a product of one-soliton matrices.

Kinks

$$\Phi(z,t) = 4\arg\left[\sqrt{p}\exp\left(\frac{i\varphi_0(\chi)}{4}\right) + \frac{i}{\sqrt{p}}\exp\left(\frac{i\varphi_0(\chi+K)}{4}\right)\right],$$

$$\varphi_0(\chi) = \pi - 2\mathrm{am}(\chi,k)$$

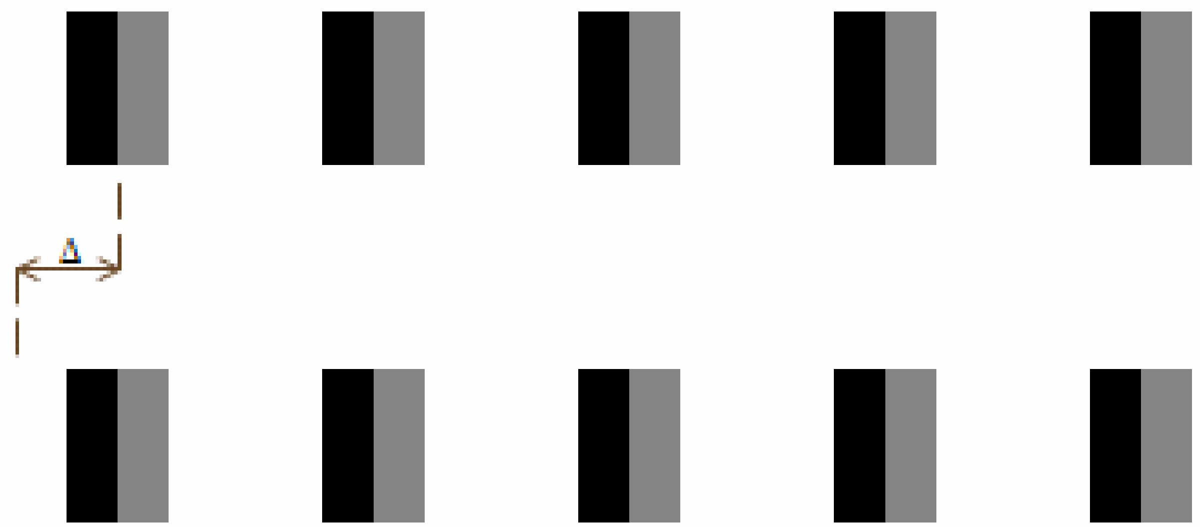
$$\chi = z/k,$$

$$p = \frac{1}{\sqrt{\mathrm{dn}\chi}}\exp\left[\frac{(1+k')\chi}{2} - \varepsilon\sqrt{k'}t\right]c,$$ $c > 0 = \mathrm{const.}$

the width of kink: $d \propto 2k/(1+k')$

kink velocity: $V = 2\varepsilon\sqrt{k'}/(1+k')$

$\varepsilon = \pm 1$



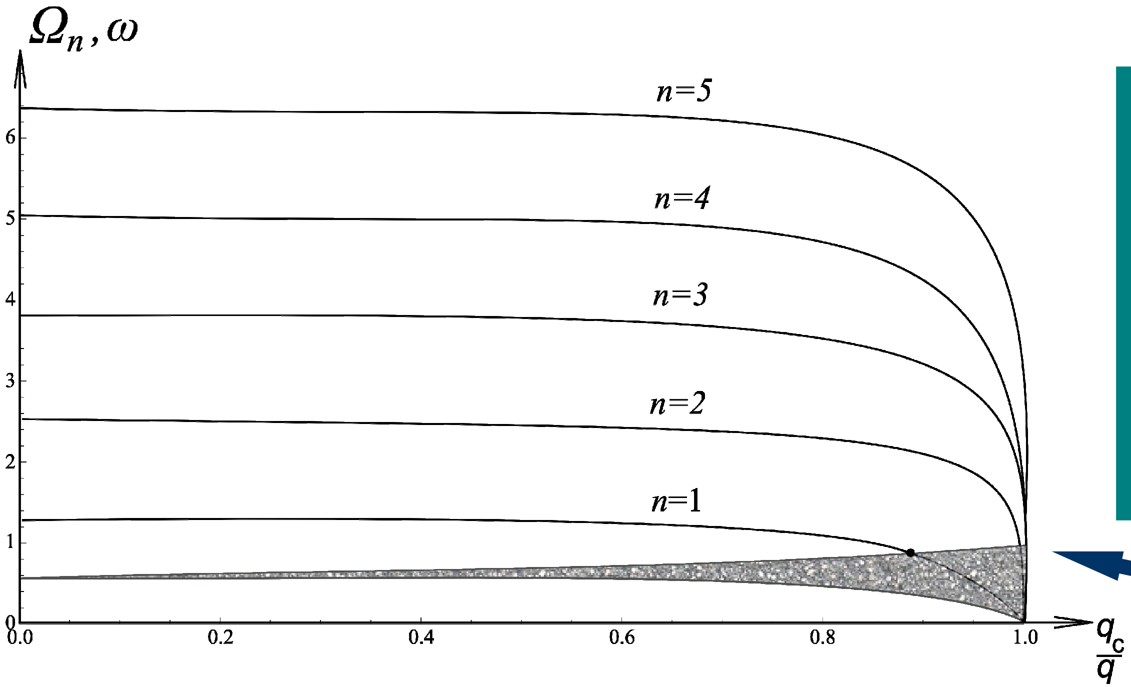
Translations of the walls can be determined by magneto-optic techniques

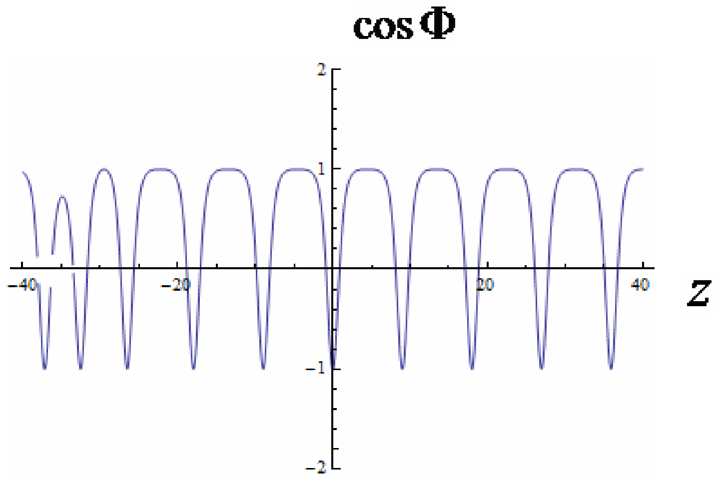
Breathers (bound state of two kinks) in helimagnet

$$\Phi(z,t)=\varphi_0(\chi+\Delta/2)+4\,\mathrm{Arctg}\left[\mathrm{tg}\left(\frac{\theta}{2}\right)\left(\frac{|m_1|^2-|m_2|^2}{|m_1|^2+|m_2|^2}\right)\right],\quad \theta=\mathrm{am}\mu+\mathrm{am}\mu^*,\quad \Delta=-4\,\mathrm{Re}\,\mu$$

$$\mathbf{m}(\chi,t\,|\,\mu)=\Psi_{\Delta/2}(\chi,t,\mu)\mathbf{c}$$

Breather has a continuous spectrum of internal oscillation, which lies low, than discrete spectrum of standing spin waves.





$$\omega=\frac{\mathrm{Re}(\mathrm{dn}\mu)}{E(k)}$$

Breather can be immobile. We suppose, that immobile breather can be detected by means of microwave power absorption on its internal oscillation frequency.

7. Spin waves (small oscillations of magnetization, gradually spreading, because of the dispersion)

boundary conditions:

$\Phi(z,t) \to \varphi_0(\chi)$ at $|z| \to \infty$

$b(u), b^*(-u^*) \neq 0\,; a(u), a^*(-u^*)$ have no zeroes,

$\Psi_\pm(u)$ are doubly periodic ($[4K, 4\mathrm{i}K']$).

To avoid singularities of background, we use new functions:

$$F_+(u) = \exp\left[\frac{\mathrm{i}}{4}(\Phi - \varphi_0)\sigma_3\right]\Psi_+(u)\Psi_0^{-1}(u), \qquad \det F_+ = a^*(-u^*),$$

$$F_-(u) = \exp\left[\frac{\mathrm{i}}{4}(\Phi - \varphi_0)\sigma_3\right]\frac{\Psi_-(u)}{a(u)}\Psi_0^{-1}(u), \qquad \det F_- = \frac{1}{a(u)},$$

Regular Riemann's problem

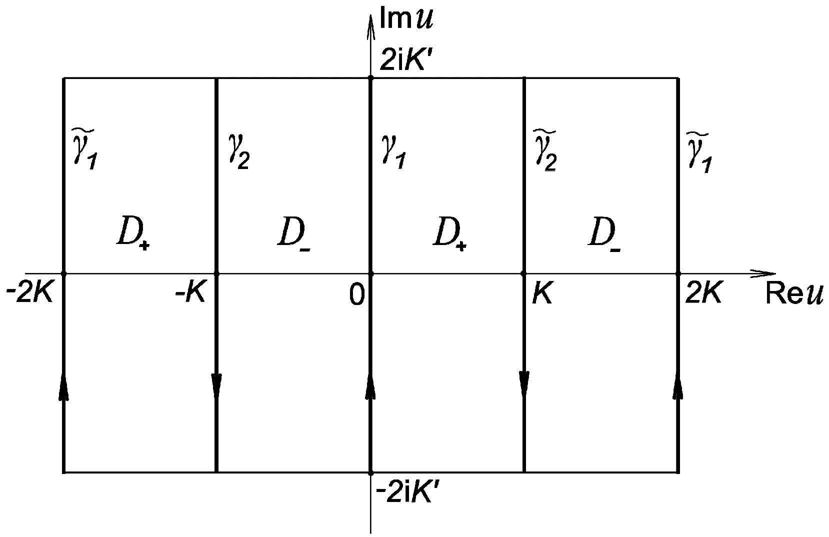
$$\boxed{F_+(u) = F_-(u)(I - G(u)), \quad G(u) = \Psi_0(u)\begin{pmatrix} 0 & b^*(-u^*) \\ b(u) & 0 \end{pmatrix}\Psi_0^{-1}(u);}$$

$$u \in \gamma = \gamma_1 \cup \gamma_2 \cup \tilde{\gamma}_1 \cup \tilde{\gamma}_2$$

reductions:

$$F_\pm(u + 2K) = \sigma_3 F_\pm(u)\sigma_3,$$
$$F_\pm^*[(u^* + 2\mathrm{i}K')^*] = \sigma_1 F_\pm(u)\sigma_1.$$

normalization: $F_\pm(-\mathrm{i}K') = I;$



SG-solution:

$$F_\pm(\mathrm{i}K') = \exp\left[\frac{\mathrm{i}}{2}(\Phi - \varphi_0)\sigma_3\right].$$

Cauchy kernel on a torus:

$$\boxed{Y(u', u) = \zeta(u' - u) - \zeta(u' + \mathrm{i}K').} \quad [4K, 4\mathrm{i}K']$$

Singular integral equation for $F_-(u)$:

$$F_-(u)\left(I - \frac{G(u)}{2}\right) = I + \frac{\text{v.p.}}{2\pi\mathrm{i}}\int_\gamma \mathrm{d}u' Y(u', u) F_-(u') G(u'). \quad (u \in \gamma)$$

At $|b(u)| << 1 \quad (u \in \gamma)$ we have found:

quasi-optical modes

$$\boxed{\begin{aligned}\Phi - \varphi_0 \approx \frac{4}{\pi}\int_{-K'}^{K'} \mathrm{d}v \Big(&\mathrm{Re}[b^*(\mathrm{i}v)\Lambda(\chi,\mathrm{i}v)\exp\big(\mathrm{i}t\,\mathrm{dn}(\mathrm{i}v,k)/k\big)] + \\ &+ \mathrm{Re}[b^*(\mathrm{i}v - K')\Lambda(\chi,\mathrm{i}v-K')\exp\big(\mathrm{i}t\,\mathrm{dn}(\mathrm{i}v-K',k)/k\big)]\Big)\end{aligned}}$$

quasi-acoustic modes

$\Lambda(\chi,u)$ - Lame function – solution of equation: $\boxed{[\partial_\chi^2 - 2k^2\mathrm{sn}^2\chi + k^2]\Lambda = -\varepsilon\Lambda}$

$$\Lambda(\chi,u) = \frac{\mathrm{i}}{2}\frac{|\sigma(\mathrm{i}K')|^2\,\sigma(K)\sigma(K+2\mathrm{i}K')}{\sigma^2(K+\mathrm{i}K')\sigma(u-\mathrm{i}K')\sigma(u+\mathrm{i}K')}\frac{\sigma(\chi+u)\sigma(\chi+2\mathrm{i}K'+u)}{\sigma(\chi-\mathrm{i}K')\sigma(\chi+\mathrm{i}K')}\times$$
$$\times\exp\big(-\eta_3(\chi+\mathrm{i}K'+u) - \chi[\zeta(u+\mathrm{i}K')+\zeta(u-\mathrm{i}K')]\big)$$

(periods of Weierstrass functions: $[2K, 4\mathrm{i}K']$)

General scheme of integration

1) Initial conditions for SG $\partial_t\Phi(t=0),\ \partial_z\Phi(t=0)$ allow to construct $a(u),\ b(u)$.

2) $a(u)$ is factorized: $\boxed{a(u)=a_{\mathrm{sol}}(u)a_0(u)}$

$a_{\mathrm{sol}}(u)$ leads to Riemann's problem, which gives $\Psi_{\pm}^{(\mathrm{sol})}(u,\chi,t)$ and $\Phi^{(\mathrm{sol})}(z,t)$.

3) $\Psi_{\pm}(u)$ are written in the form:

$$\boxed{\begin{aligned}\Psi_+(u) &= \exp\left(-\frac{\mathrm{i}(\Phi-\Phi^{(\mathrm{sol})})\sigma_3}{4}\right)F_+(u)\Psi_+^{(\mathrm{sol})}(u),\\ \Psi_-(u) &= \exp\left(-\frac{\mathrm{i}(\Phi-\Phi^{(\mathrm{sol})})\sigma_3}{4}\right)F_-(u)\Psi_-^{(\mathrm{sol})}(u)a_0(u),\end{aligned}}$$

The calculation of $F_{\pm}(u)$ leads to above mentioned
<u>regular Riemann's problem:</u>

$$\boxed{F_+(u) = F_-(u)(I-G(u)), \qquad u\in\gamma,}$$

$$G(u) = \Psi_-^{(\mathrm{sol})}(u)\begin{pmatrix} 0 & b^*(-u^*) \\ b(u) & 0 \end{pmatrix}(\Psi_-^{(\mathrm{sol})}(-u^*))^+\,\frac{(\mathrm{dn}u-1)}{\mathrm{dn}u}.$$

<u>Solution of initial boundary value problem</u> for SG:

$$\boxed{F_{\pm}(\mathrm{i}K') = \exp\left[\frac{\mathrm{i}(\Phi-\Phi^{(\mathrm{sol})})\sigma_3}{2}\right].}$$

8. Spectral expansions of integrals of motion for collective excitations of spiral structure

The main problem in obtaining conservation laws for spiral structure is to separate the contributions from inhomogeneous ground state.

All difficulties are overcame, if independent of time function $a(u)$ is used as generating functional for the integrals of motion. We have found spectral expansions for conservation laws by means of dispersion relation on a torus for the function $\ln a(u)$.

We define an energy of collective excitations as difference between complete energy of system and the energy of ground state:

$$H=\int_{-K'}^{K'}\mathrm{d}v\left[\varepsilon_{\mathrm{a}}(v)n_{\mathrm{a}}(v)+\varepsilon_{\mathrm{opt}}(v)n_{\mathrm{opt}}(v)\right]-\frac{8}{k}\left(\sum_{p=1}^{2M}\mathrm{Re}\left[Z(v_p)+\frac{2\eta_1 v_p}{K}\right]+2\sum_{s=1}^{N}\mathrm{Re}\left[Z(\mu_s)+\frac{2\eta_1\mu_s}{K}\right]\right)+2q\mathrm{am}(\Delta,k).$$

Spectrum and density of quasi-optical and quasi-acoustic spin wave modes

Contributions of kinks and breathers

Compression of structure, because of kinks and breathers

$$\mathrm{Re}\left(\sum_{p=1}^{2M}v_p+2\sum_{s=1}^{N}\mu_s\right)=-\frac{\Delta}{2}$$

$$\varepsilon_{\mathrm{a}}(v)=k'^2\,\mathrm{sn}^2(v+K',k'),\quad n_{\mathrm{a}}(v)=\frac{4}{\pi k}\ln\left(1+|b(-K+\mathrm{i}v)|^2 e^{2\eta_1\Delta}\right)>0;$$

$$\varepsilon_{\mathrm{opt}}(v)=\frac{1}{\mathrm{sn}^2(v+K',k')},\quad n_{\mathrm{opt}}(v)=-\frac{4}{\pi k}\ln\left(1-|b(\mathrm{i}v)|^2\right)>0.$$

Field momentum of collective excitations:

$$P=\int_{-K'}^{K'}\mathrm{d}v\left[p_{\mathrm{a}}(v)n_{\mathrm{a}}(v)+p_{\mathrm{opt}}(v)n_{\mathrm{opt}}(v)\right]-\frac{8}{k}\left(\sum_{p=1}^{2M}\mathrm{Im}(\mathrm{dn}\,\nu_p)+2\sum_{s=1}^{N}\mathrm{Im}(\mathrm{dn}\,\mu_s)\right),$$

Spin wave momentums are distinct from quasi-momentums of Lame function:

$$p_{\mathrm{a}}(v)=k'\,\mathrm{dn}(v+K',k')\,\mathrm{cn}(v+K',k'),\qquad p_{\mathrm{opt}}(v)=-\frac{\mathrm{dn}(v+K',k')\,\mathrm{cn}(v+K',k')}{\mathrm{sn}^2(v+K',k')}.$$

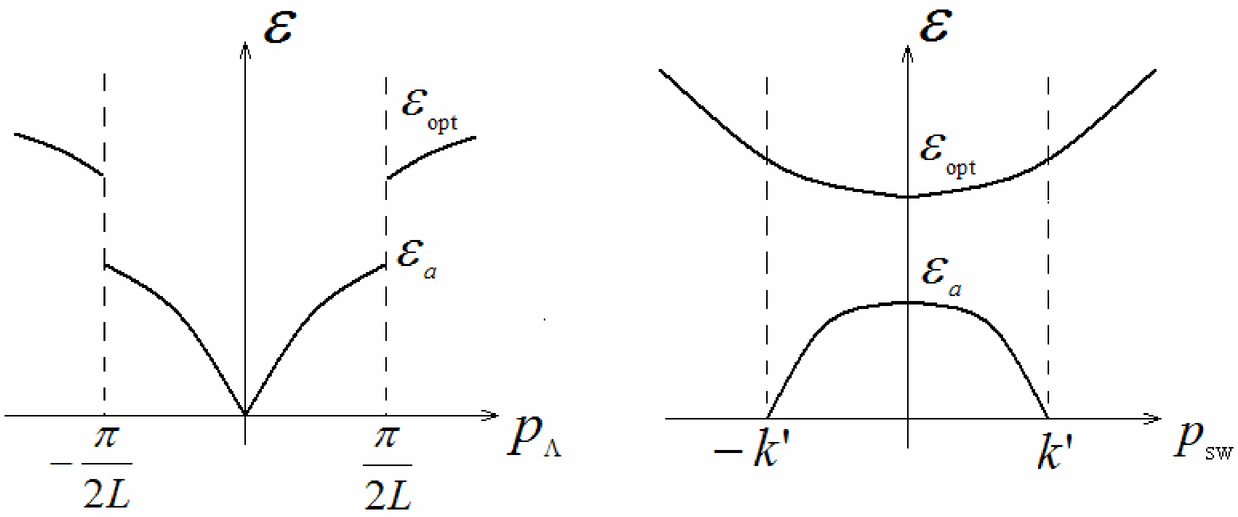
The contributions of solitons and spin waves to the conservation laws are completely separated from each other.

Usually, the dispersion laws of spin waves are written in terms of quasi-momentum of Lame function (left fig.) and <u>they are transcendental</u>.

In terms of spin wave momentum (right fig.), the dispersion laws of spin wave modes <u>are algebraic</u>.

$$\varepsilon_{\mathrm{a}}(p_{\mathrm{a}})=\frac{1}{2}\left(1+k'^2-\sqrt{k^4+4p_{\mathrm{a}}^2}\right), \qquad |p_{\mathrm{a}}|\le k';$$

$$\varepsilon_{\mathrm{opt}}(p_{\mathrm{opt}})=\frac{1}{2}\left(1+k'^2+\sqrt{k^4+4p_{\mathrm{opt}}^2}\right), \quad -\infty<p_{\mathrm{opt}}<\infty.$$



Conclusion

- We propose analytic description of nonlinear collective excitations in spiral structure of magnets without inversion center in the framework of sine-Gordon model. We can find exact solutions for solitons and spin waves with an arbitrary initial distribution of magnetization in helimagnet.

- We have shown, that solitons lead to macroscopic shift of spiral structure. This shift can be detected by magneto-optics.

- Breathers in spiral structure can be detected from the resonanse microwave power absorption on the frequency of their internal oscillations.

- Spectral expansions for integrals of motion, including soliton and spin wave contributions, are found.

Thank you for attention