Interval analysis and robotics

Luc Jaulin

Labsticc, IHSEV, ENSTA-Bretagne
2, rue François Verny,
29200, Brest, France
luc.jaulin@ensta-bretagne.fr

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When dealing with complex mobile robots, we often have to solve a huge set of nonlinear equations. They may be related to some measurements collected by sensors, to some prior knowledge on the environment or to the differential equations describing the evolution of the robot. For a large class of robots these equations are uncertain, enclose many unknown variables, are strongly nonlinear and should be solved very quickly. Hopefully, the number of these equations is generally much larger than the number of variables. We can assume that the system to be solved has the following form

$$\left\{ \begin{array}{c} f_i\left(x, y_i\right) = 0, \\ x \in \mathbb{R}^n, \quad y_i \in [y_i] \subset \mathbb{R}^{p_i}, \\ i \in \{1, \ldots, m\}. \end{array} \right. \tag{1}$$

The vector $x \in \mathbb{R}^n$ is the vector of unknown variables, the vector $y_i \in \mathbb{R}^{p_i}$ is the ith data vector (which is approximately known) and $f_i : \mathbb{R}^n \times \mathbb{R}^{p_i} \to \mathbb{R}$ is the ith function. The box $[y_i]$ is a small box of $\mathbb{R}^n$ that takes into account some uncertainties on y_i. Here, we assume that the number of equations m is much larger that the number of unknown variables n (otherwise, the method will not be able to provide accurate results). Typically, we could have $n = 1000$ and $m = 10000$. In order to provide a fast polynomial algorithm able to find a box $[x]$ that encloses all feasible x, we shall associate, to each equation $f_i\left(x, y_i\right) = 0$, a contractor $C_i : \mathbb{IR}^n \to \mathbb{IR}^n$ that narrows the box $[x]$ without removing any value for x consistent with the ith equation. Such a contractor can be obtained using interval computations [1]. Then we iterate each contractor until no more contraction can be performed. An illustration of the procedure is Figure 1, where the sequence of contractors C_1, C_2, C_3, C_1, C_2, $C_3 \ldots$ is applied. Note that the first contractor C_1 was able to contract the initial box $[x] = [-\infty, \infty]^2$ to the box containing the thick circle.

As an example, we shall consider the SLAM (Simultaneous localization and map building) problem asking whether it is possible for an autonomous robot

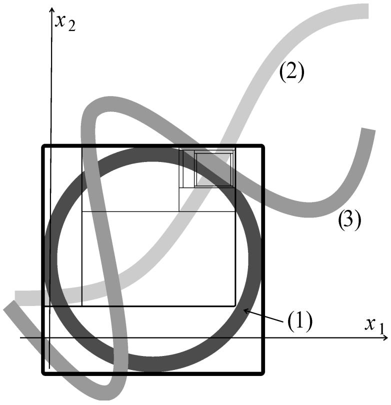

Figure 1: Illustration of the propagation process

to move in an unknown environment and build a map of this environment while simultaneously using this map to compute its location. It is shown in [2] that the general SLAM problem can be cast into the form (1). The corresponding system is strongly nonlinear and classical non-interval methods cannot to deal with this type of problems in a reliable way. The efficiency of the approach will be illustrated on a two-hour experiment where an actual underwater robot is involved. This four-meter long robot build by the GESMA (Groupe d'étude sous-marine de l'Atlantique) is equipped with many sensors (such as sonars, Loch-Doppler, gyrometers, ...) which provide the data. The algorithm is able to provide an accurate envelope for the trajectory of the robot and to compute sets which contain some detected mines in less than one minute. Other examples involving underwater robots and sailboat robots will also be presented.

References:

[1] R.E. Moore, R.B. Kearfott, M.J. Cloud, *Introduction to Interval Analysis*, SIAM, Philadelphia, 2009.

[2] L. Jaulin, A nonlinear set-membership approach for the localization and map building of an underwater robot using interval constraint propagation, *IEEE Transaction on Robotics*, 25 (2009), No. 1, pp. 88–98.