Decision making under interval uncertainty

Vladik Kreinovich

Department of Computer Science
University of Texas at El Paso
El Paso, TX 79968, USA
vladik@utep.edu

Keywords: interval uncertainty, decision making, utility theory, p-boxes, modal intervals, symmetries, control

To make a decision, we must:

- find out the user's preference, and
- help the user select an alternative which is the best – according to these preferences.

A general way to describe user preferences is via the notion of *utility* (see, e.g., [7]): we select a very bad alternative A_0 and a very good alternative A_1; utility $u(A)$ of an alternative A if then defined as the probability p for which A is equivalent to the lottery in which we get A_1 with probability p, and A_0 otherwise. One can prove that utility is determined uniquely modulo linear re-scaling (corresponding to different choices of A_0 and A_1), and that the utility of a decision with probabilistic consequences is equal to the expected utility of these consequences.

Once the utility function $u(d)$ is elicited, we select the decision $d_{\rm opt}$ with the largest utility $u(d)$. Interval techniques can help in finding the optimizing decision; see, e.g., [4].

Often, we do not know the exact probability distribution, so we need to extract, from the sample, the characteristics of a distribution which are most appropriate for decision making. We show that, under reasonable assumptions, we should select moments and cumulative distribution function (cdf). Based on a finite sample, we can only find bounds on these characteristics, so we need to deal with bounds (intervals) on moments [6] and bounds on cdf [1] (a.k.a. p-boxes).

Once we know intervals $[\underline{u}(d), \overline{u}(d)]$ of possible values of utility, which decision shall we select? We can simply select a decision d_0 which *may* be optimal, i.e., for which $\overline{u}(d_0) \geq \max_d \underline{u}(d)$, but there are usually many such decisions; which of them should be select? It is reasonable to assume that this selection should not depend on linear re-scaling of utility; under this assumption, we get Hurwicz optimism-pessimism criterion $\alpha \cdot \overline{u}(d) + (a - \alpha) \cdot \underline{u}(d) \to \max$ [7]. The next question is how to select α: interestingly, e.g., too optimistic values $(\alpha > 0.5)$ do not lead to good decisions.

In some situations, it is difficult to elicit even interval-valued utilities. In many such situations, there are reasonable symmetries which can be used to make a decision; see, e.g., [5]. We show how this method works on the example of selecting a location for a meteorological tower [3].

Finally, while optimization problems are ubiquitous, sometimes, we need to go beyond optimization: e.g., we need to make sure that the system is *controllable* for all disturbances within a given range. In such problems, modal intervals [2] naturally appear. In more complex situations, we need to go beyond modal intervals, to more general Shary's classes.

References:

[1] S. Ferson, V. Kreinovich, J. Hajagos, W. Oberkampf, L. Ginzburg, *Experimental Uncertainty Estimation and Statistics for Data Having Interval Uncertainty*, Sandia National Laboratories, 2007, Publ. 2007-0939.

[2] E. Gardeñes et al., Modal intervals, *Reliable Computing*, 7 (2001), pp. 77–111.

[3] A. Jaimes, C. Tweedie, V. Kreinovich, M. Ceberio, Scale-invariant approach to multi-criterion optimization under uncertainty, with applications to optimal sensor placement, in particular, to sensor placement in environmental research, *International Journal of Reliability and Safety*, 6 (2012), No. 1–3, pp. 188–203.

[4] R.E. Moore, R.B. Kearfott, M.J. Cloud, *Introduction to Interval Analysis*, SIAM, Philadelphia, 2009.

[5] H.T. Nguyen, V. Kreinovich, *Applications of continuous mathematics to computer science*, Kluwer, Dordrecht, 1997.

[6] H.T. Nguyen, V. Kreinovich, B. Wu, G. Xiang, *Computing Statistics under Interval and Fuzzy Uncertainty*, Springer Verlag, 2012.

[7] H. Raiffa, *Decision Analysis*, McGraw-Hill, Columbus, Ohio, 1997.