Properties and Estimations of Parametric AE-Solution Sets

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Consider linear systems $A(p)x = b(p)$, where the elements of the matrix and right-hand side vector are linear functions of uncertain parameters varying within given intervals, $p_i \in [p_i]$, $i = 1, \dots, k$. Such systems are common in many engineering analysis or design problems, control engineering, robust Monte Carlo simulations, etc., where there are complicated dependencies between the model parameters which are uncertain. Various solution sets to a parametric linear system can be defined depending on the way the parameters are quantified by the existential and/or the universal quantifiers. We are interested in the parametric AE-solution sets, which are defined by universally and existentially quantified parameters, and the former precede the latter. For two disjoint sets of indexes $\mathcal{E}$ and $\mathcal{A}$, such that $\mathcal{E} \cup \mathcal{A} = \{1, \dots, k\}$,

$$\begin{aligned}\Sigma^p_{AE} &= \Sigma(A(p_{\mathcal{A}}, p_{\mathcal{E}}), b(p_{\mathcal{A}}, p_{\mathcal{E}}), [p]) \\ &:= \{x \in \mathbf{R}^n \mid (\forall p_{\mathcal{A}} \in [p_{\mathcal{A}}])(\exists p_{\mathcal{E}} \in [p_{\mathcal{E}}])(A(p)x = b(p))\}.\end{aligned}$$

Parametric AE-solution sets generalize the parametric united solution set and the nonparametric AE-solution sets.

In this talk we present three types of characterizations for the parametric AE-solution sets: set-theoretic characterization, characterization in form of interval inclusions and characterization by Oettli-Prager-type absolute-value inequalities. The focus of the characterizations is on how to obtain explicit description of a parametric AE-solution set in the form of Oettli-Prager-type inequalities. The description is explicit for some classes of parametric AE-solution sets and in the general case can be obtained by a Fourier-Motzkin-type algorithmic procedure eliminating the existentially quantified parameters.

The characterizations of parametric AE-solution sets inspire proving various properties of these solution sets and designing some numerical methods for their outer or inner estimation. We will present some important inclusion relations between classes of parametric AE-solution sets, where the relations are determined by the type of the parameter dependencies. Various other properties like the shape of a parametric AE-solution set and some criteria for nonempty and bounded solution set will be also discussed. Special consideration is provided for the parametric tolerable solution set

$$\begin{aligned}\Sigma^p_{tol} &= \Sigma(A(p_{\mathcal{A}}), b(p_{\mathcal{E}}), [p]) \\ &:= \{x \in \mathbf{R}^n \mid (\forall p_{\mathcal{A}} \in [p_{\mathcal{A}}])(\exists p_{\mathcal{E}} \in [p_{\mathcal{E}}])(A(p_{\mathcal{A}})x = b(p_{\mathcal{E}}))\}\end{aligned}$$

and for the parametric controllable solution set

$$\begin{aligned}\Sigma^p_{cont} &= \Sigma(A(p_{\mathcal{E}}), b(p_{\mathcal{A}}), [p]) \\ &:= \{x \in \mathbf{R}^n \mid (\forall p_{\mathcal{A}} \in [p_{\mathcal{A}}])(\exists p_{\mathcal{E}} \in [p_{\mathcal{E}}])(A(p_{\mathcal{E}})x = b(p_{\mathcal{A}}))\}.\end{aligned}$$

Some numerical methods for outer and inner estimations of parametric AE-solution sets will be also presented. The properties of these methods for estimating the parametric tolerable and the parametric controllable solution sets are compared. We show that in some cases the parametric approach provides a more efficient solution for some nonparametric problems than the existing nonparametric approaches. Numerical examples accompanied by graphic representations will illustrate the solution sets and their properties or the numerical methods and their properties. Some of the properties (or methods) are new, most of them generalize known properties (or methods) for nonparametric AE-solution sets, studied by I. Sharaya, S. Shary and others. Presenting some first results about parametric AE-solution sets, the talk will also outline some open problems and directions of possible further research.