Deterministic global optimization using the Lipschitz condition

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In this lecture, the global optimization problem of a multidimensional function satisfying the Lipschitz condition with an unknown Lipschitz constant over a multi-dimensional box is considered. It is supposed that the objective function can be "black box", multiextremal, and non-differentiable. It is also assumed that evaluation of the objective function at a point is a time-consuming operation. Many algorithms for solving this problem have been discussed in the literature (see [1–12] and references given therein). They can be distinguished, for example, by the way of obtaining an information about the Lipschitz constant and by the strategy used to explore the search domain.

Different exploration techniques based on various adaptive partition strategies are analyzed. The main attention is dedicated to diagonal algorithms, since they have a number of attractive theoretical properties and have proved to be efficient in solving applied problems. In these algorithms, the search box is adaptively partitioned into sub-boxes and the objective function is evaluated only at two vertices corresponding to the main diagonal of the generated sub-boxes.

It is demonstrated that the traditional diagonal partition strategies do not fulfil the requirements of computational efficiency because of executing many redundant evaluations of the objective function. A new adaptive diagonal partition strategy that allows one to avoid such computational redundancy is described. Some powerful multidimensional global optimization algorithms based on the new strategy are introduced. Results of extensive numerical experiments performed to test the methods proposed demonstrate their advantages with respect to diagonal algorithms in terms of both number of trials of the objective function and qualitative analysis of the search domain, which is characterized by the number of generated boxes. Finally, problems with Lipschitz first derivatives are considered and connections between the Lipschitz global optimization and interval analysis global optimization are discussed.

References:

[1] L.G. Casado, I. Garcia, Ya.D. Sergeyev, Interval algorithms for finding the minimal root in a set of multiextremal non-differentiable one-dimensional functions, *SIAM J. Scientific Computing*, 24 (2002), No. 2, pp. 359–376.

[2] Yu.G. Evtushenko, M.A. Posypkin, An application of the nonuniform covering method to global optimization of mixed integer nonlinear problems, *Comput. Math. Math. Phys.*, 51 (2011), No. 8, pp. 1286–1298.

[3] R. Horst and P.M. Pardalos, eds., *Handbook of Global Optimization*, Kluwer Academic Publishers, Dordrecht, 1995.

[4] D.E. Kvasov, Ya.D. Sergeyev, Univariate geometric Lipschitz global optimization algorithms, *NACO*, 2 (2012), No. 1, 69–90.

[5] D. Lera, Ya.D. Sergeyev, An information global minimization algorithm using the local improvement technique, *J. Global Optimization*, 48 (2010), No. 1, 99–112.

[6] J. Pintér, *Global Optimization in Action*, Kluwer, Dordrecht, 1996.

[7] Ya.D. Sergeyev, D.E. Kvasov, *Diagonal Global Optimization Methods*, FizMatLit, Moscow, 2008.

[8] Ya.D. Sergeyev, D.E. Kvasov, Lipschitz global optimization, *Wiley Encyclopaedia of Operations Research and Management Science*, 4 (2011), pp. 2812–2828.

[9] Ya.D. Sergeyev, D.E. Kvasov, Global search based on efficient diagonal partitions and a set of Lipschitz constants, *SIAM J. Optimization*, 16 (2006), No. 3, pp. 910–937.

[10] Ya.D. Sergeyev, P. Pugliese, D. Famularo, Index information algorithm with local tuning for solving multidimensional global optimization problems with multiextremal constraints, *Math. Programming*, 96 (2003), No. 3, pp. 489–512.

[11] R.G. Strongin, Ya.D. Sergeyev, Global optimization with non-convex constraints: Sequential and parallel algorithms, Kluwer, Dordrecht, 2000.

[12] A.A. Zhigljavsky, A. Žilinskas, *Stochastic Global Optimization, Springer*, New York, 2008.