New directions in interval linear programming

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Linear programming is undoubtedly one of the most frequently used technique in problem solving. Since real life data are often not known precisely due to measurement errors, estimations and other kinds of uncertainties, we have to reflect it in the theory of linear programming. Modeling such uncertainties by intervals gives rise to the research area called interval linear programming [1,2]. Herein, we suppose that interval domains of uncertain quantities are a priori given, and the aim is to calculate verified results giving rigorous enclosures (or other types of answers) valid for all possible realizations of interval data.

There are many problems regarding interval linear programming, such as verifying feasibility, (un)boundedness or optimality for some or for all realizations of interval quantities; some of them are polynomially solvable, but the others are NP-hard. However, there are two main directions of determining (enclosing) the optimal value range and the optimal solution set. While the former was intensively studied in the past and many results concerning computational complexity and methods are available, there is still lack of theory and practical methods for the latter. Rigorously and tightly enclosing the optimal solution set is the most challenging problem in this subject. Traditional approaches were based on the so called basis stability, meaning that there is a basis being optimal for each realization of intervals. Under basis stability, the problems can be solved very efficiently. Checking this property may be computationally expensive in general, but strong sufficient conditions exist. The problem is, however, that in many situations (e.g. under basis degeneracy), the problem is not basis stable even for tiny intervals. Overcoming the non-basis stability is remaining to be an important but difficult problem.

In the talk, we survey the known results and present recent developments as well. We focus on the computational complexity, methods and other aspects of enclosing the optimal value range and the optimal solution set. We discuss applications of this technique in diverse areas. Besides many real-world optimization problems (in economics, environmental management, logistics, ...), interval linear programming may also serve as a supporting tool for linear relaxation in constraint programming and global optimization, in matrix games with inexact data or in statistics in linear regression on uncertain data by using L_1 or L_∞ norm. Sensitivity analysis, frequently used in economical operations research, can be extended from the traditional one-parameter case to the case with multiple parameters situated in diverse positions. Eventually, we mention some open problems and challenges for the future research.

References:

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