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Maximizing stability degree of interval systems using coefficient method

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Goals and Objectives

Objective:

Parametric synthesis and maximizing stability degree of interval automatic control system based on coefficient estimations of system quality indices.

Goals:

- to define interval system characteristic polynomial coefficients;
- to form interval inequalities system based on coefficient estimations of system quality indices;
- to choose system solution providing stability degree maximization;
- to check the synthesis results on a numerical example.

Block diagram

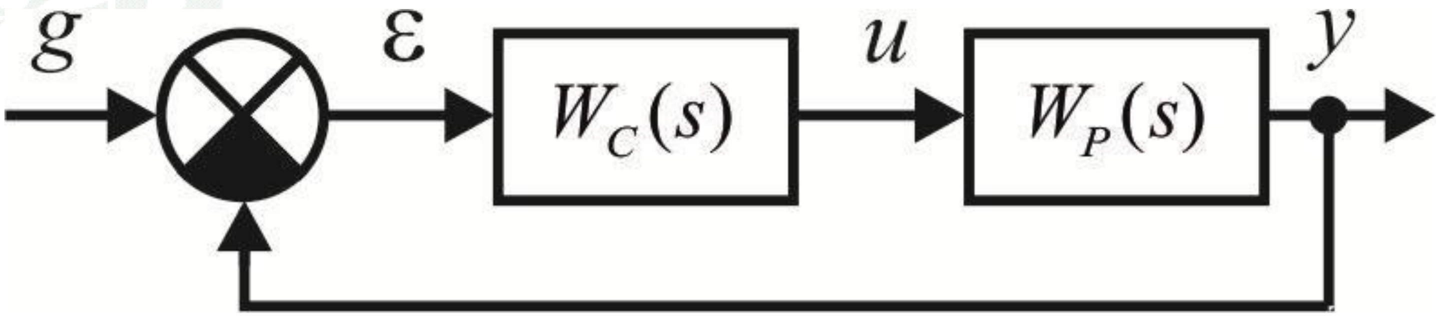

Fig. 1 Control system block diagram

$$W_P(s) = \frac{k_P}{\sum_{i=0}^{z} \left[\underline{d}_i, \overline{d}_i\right] s^i}$$ - plant transfer function

$$W_C(s) = \frac{k_1 s + k_0}{s}$$ - controller transfer function

Problem formulation

System characteristic polynomial

$$A(s) = s\sum_{i=0}^{z}\left[\underline{d}_i, \overline{d}_i\right]s^i + k_P\left(k_1 s + k_0\right) \qquad (1)$$

$$A(s) = \left[\underline{a}_n, \overline{a}_n\right]s^n + \left[\underline{a}_{n-1}, \overline{a}_{n-1}\right]s^{n-1} + \ldots + \left[\underline{a}_0, \overline{a}_0\right], \quad a_n > 0 \qquad (2)$$

Goal: to define controller coefficients providing maximization of transfer function roots location from the imaginary axis on a complex plane, that ensure interval system stability degree maximization.

Quality indices coefficient estimation method

In order that all the characteristic polynomial roots located on the left side of the vertical line going through the point $(-\eta, j0)$, $0 < \eta < \infty$, it is sufficient the following conditions to be held

$$
\begin{cases}
\dfrac{a_{i-1}a_{i+2}}{\left[a_i - a_{i+1}(n-i-1)\eta\right]\left[a_{i+1} - a_{i+2}(n-i-2)\eta\right]} < \lambda^*, \quad i = \overline{1,\, n-2}; & (3) \\
a_l - a_{l+1}(n-l-1)\eta \geq 0, \quad l = \overline{1,\, n-1}; & (4) \\
a_0 - a_1\eta + \dfrac{2a_2\eta^2}{3} \geq 0. & (5)
\end{cases}
$$

where $\lambda^* \approx 0.465$ - stability index;

η - stability degree;

a_i, a_l - characteristic polynomial coefficients.

Quality indices coefficient estimation method

Introduce the designations:

$$\lambda_i(\eta) = \frac{a_{i-1}a_{i+2}}{\left[a_i - a_{i+1}(n-i-1)\eta\right]\left[a_{i+1} - a_{i+2}(n-i-2)\eta\right]}, \quad k = \overline{1, n-2}; \qquad (6)$$

$$f_l(\eta) = a_l - a_{l+1}(n-l-1)\eta, \;\; l = \overline{1, n-1}; \qquad (7)$$

$$g(\eta) = a_0 - a_1\eta + \frac{2a_2\eta^2}{3}. \qquad (8)$$

Introduce in consideration the concept of maximum stability degree according to the stability index λ_i, $i = \overline{1, n-2}$ and designate it η_i.

Quality indices coefficient estimation method

$$\begin{cases} \lambda_i\left(\eta,\bar{\mathrm{k}}\right)=\lambda^*, & i\in\overline{1,\ n-2}; \\ \lambda_j\left(\eta,\bar{\mathrm{k}}\right)<\lambda^*, & j=\overline{1,\ n-2};\ \ j\neq i \\ f_l\left(\eta,\bar{\mathrm{k}}\right)\geq 0, & l=\overline{1,\ n-1}; \\ g\left(\eta,\bar{\mathrm{k}}\right)\geq 0. & \end{cases} \qquad (9)$$

$(\bar{k})$ - tunable controller parameters vector.

It is necessary to solve the given equation system $(n-2)$ times and define maximal η_i on each step and after that choose the maximal value among them $\eta^* = \max \eta_i$.

$$\eta^*_{\max} = \max_{\bar{k}} \eta^*$$

All the roots real parts of the synthesized system will always lie on the left side of the vertical line going through the point $(-\eta, j0)$, $0<\eta<\infty$.

$\eta_{real} > \eta^*_{\max} \Rightarrow \eta^*_{\max}$ - lower estimate of maximum stability degree

Interval extension of quality indices coefficient estimation method

$$\begin{cases} \dfrac{\overline{a_{i-1}(\bar{k})}\,\overline{a_{i+2}(\bar{k})}}{\left[\underline{a_i(\bar{k})} - a_{i+1}(\bar{k})\left(n-i-1\right)\eta\right]\left[a_{i+1}(\bar{k}) - \overline{a_{i+2}(\bar{k})}\left(n-i-2\right)\eta\right]} = \lambda^*, \quad i = \overline{1,\, n-2}; \\[2ex] \dfrac{\overline{a_{i-1}(\bar{k})}\,\overline{a_{i+2}(\bar{k})}}{\left[\underline{a_i(\bar{k})} - a_{i+1}(\bar{k})\left(n-i-1\right)\eta\right]\left[a_{i+1}(\bar{k}) - \overline{a_{i+2}(\bar{k})}\left(n-i-2\right)\eta\right]} < \lambda^*, \quad j = \overline{1,\, n-2},\ j \neq i; \\[2ex] \underline{a_l(\bar{k})} - \overline{a_{l+1}(\bar{k})}\left(n-l-1\right)\eta \geq 0, \quad l = \overline{1,\, n-1}; \\[2ex] \underline{a_0(\bar{k})} - \overline{a_1(\bar{k})}\eta + \dfrac{2\underline{a_2(\bar{k})}\eta^2}{3} \geq 0. \end{cases} \qquad (10)$$

It's defined that for requirements (10) fulfillment coefficients $a_{i+1}(\bar{k})$ and $a_{j+1}(\bar{k})$ can take both minimal and maximal values. Therefore system of inequalities (10) should be solved for both limits of the given coefficients.

Interval extension of quality indices coefficient estimation method

For the verification of conditions (10) and correct synthesis it is necessary to consider the following polynomials:

$$D_1(s) = \overline{a_0} + \underline{a_1}s + \overline{a_2}s^2 + \overline{a_3}s^3 + \underline{a_4}s^4 + \overline{a_5}s^5 + \overline{a_6}s^6 + \ldots,$$

$$D_2(s) = \overline{a_0} + \overline{a_1}s + \underline{a_2}s^2 + \overline{a_3}s^3 + \overline{a_4}s^4 + \underline{a_5}s^5 + \overline{a_6}s^6 + \ldots,$$

$$D_3(s) = \underline{a_0} + \overline{a_1}s + \overline{a_2}s^2 + \underline{a_3}s^3 + \overline{a_4}s^4 + \overline{a_5}s^5 + \underline{a_6}s^6 + \ldots,$$

$$D_4(s) = \underline{a_0} + \overline{a_1}s + \underline{a_2}s^2 + \overline{a_3}s^3 + \underline{a_4}s^4 + \overline{a_5}s^5 + \underline{a_6}s^6 + \ldots,$$

+4 Kharitonov polynomials

Methodics

- to obtain closed-loop interval system characteristic polynomial with controller parameters;
- to form inequalities system with regard to imposed requirements to the system and interval characteristic polynomial coefficients;
- to obtain controller coefficients on the assumption of system stability degree maximization;
- to check obtained results by means of root location domains plot.

Numerical example

Consider the plant with the transfer function

$$W_P(s) = \frac{1}{[0.0004;0.0007]s^3 + [0.055;0.057]s^2 + [0.61;0.64]s + [0.9;1.1]} \tag{11}$$

It is necessary to choose controller parameters

$$W_C(s) = \frac{k_1 s + k_0}{s} \tag{12}$$

providing maximizing system stability degree.

Along with the system stability degree maximization guaranteeing, it is also possible to provide specified control accuracy. Taking into account the value of quality factor for speed error elimination

$$k_0 b_0 = D a_0 \Rightarrow k_0 = \frac{D a_0}{b_0}$$

Numerical example

On the basis of (10) and (11) interval system characteristic polynomial has the form

$$\left[d_3\right]s^4+\left[d_2\right]s^3+\left[d_1\right]s^2+\left(b_0k_1+\left[d_0\right]\right)s+b_0k_0=0$$

According to accuracy requirements imposed to the system $k_0=\dfrac{Da_0}{b_0}=10.$

$$\eta^{*}_{\max}=2.1354$$

$$W_C(s)=\frac{3.962s+10}{s}$$

$$\left[0.0004;0.0007\right]s^4+\left[0.055;0.057\right]s^3+\left[0.61;0.64\right]s^2+\left[4.862;5.062\right]s+9=0$$

Numerical example

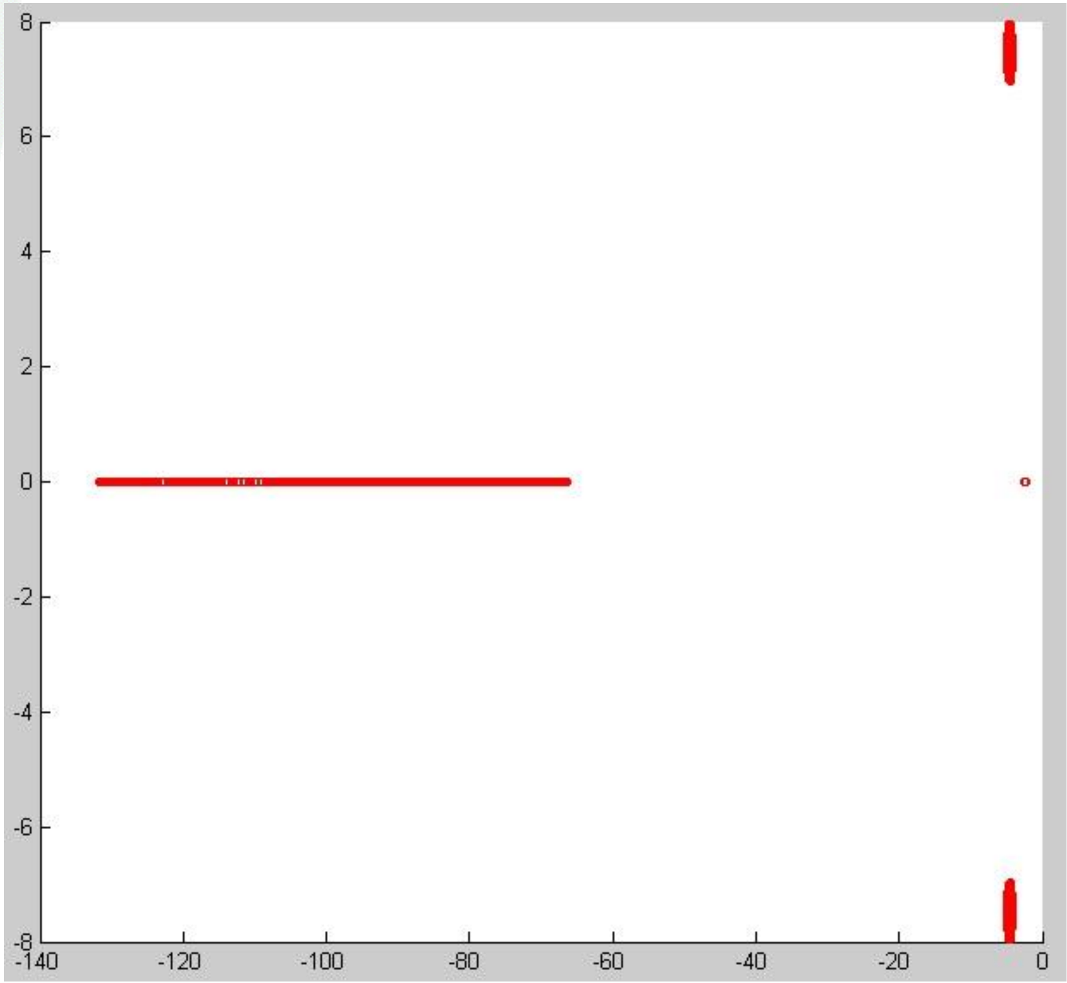

Fig. 1. Interval system poles location

Numerical example

Estimated stability degree bound:

$$\eta^*_{\max} = 2.1354$$

Real stability degree value:

$$\eta_{real} = 2.271$$

Results

- Interval systems stability degree maximizing conditions based on coefficients methods are formulated.
- PI-controller synthesis methodic providing maximizing stability degree of interval system is developed.
- Numerical example is considered.
- Synthesis results analysis on the basis of root location domain plot is conducted.

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Thank you for attention!

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