LAVRENTYEV INSTITUTE OF HYDRODYNAMICS OF SB RAS

NOVOSIBIRSK STATE UNIVERSITY

THE SECOND RUSSIA-JAPAN WORKSHOP

MATHEMATICAL ANALYSIS OF FRACTURE PHENOMENA FOR ELASTIC STRUCTURES AND ITS APPLICATIONS

20TH CONFERENCE OF CONTINUUM MECHANICS FOCUSING ON SINGULARITIES (CoMFoS20)

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ABSTRACTS

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The mathematical foundation of fracture mechanics has seen considerable advances in the last years. This field of study covers a big variety of exciting topics, including propagation of cracks, equilibrium of structures with thin inclusions in the presence of delaminations, frictional contact problems, inverse and control problems. The aim of the workshop "Mathematical analysis of fracture phenomena for elastic structures and its application" is to bring together researchers working on different aspects of these issues. The workshop provides a platform for researchers to communicate, discuss, and exchange ideas under the common theme of fracture phenomena.

CoMFoS was initiated in 1995 under the auspices of the activity group "Continuum Mechanics Focusing on Singularities (CoMFoS)" of the Japan Society for Industrial and Applied Mathematics (JSIAM). From April 2010, the activity group CoMFoS was renamed "Mathematical Aspects of Continuum Mechanics (MACM)". This is the 20th conference of CoMFoS and will be held under the co-sponsorship of the Japan-Russia Research Cooperative Program.

The Workshop and CoMFoS topics:

- elasticity, plasticity
- modeling of composite materials
- fracture mechanics
- study of mathematical models for solids with defects
- asymptotic and multiscale analysis
- optimal shape design
- inverse problems

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A VARIATIONAL APPROACH TO MODELING THERMOELASTIC PROBLEMS

Sayahdin Alfat[1,2] and Masato Kimura[2]

[1] *Physics Education Department, Halu Oleo University, Kendari, Sulawesi Tenggara, Indonesia*

[2] *Division of Mathematical and Physical Sciences, Kanazawa University, Ishikawa, Japan*

A temperature change in an elastic medium results in its deformation. Conversely, a deformation induces a change in temperature. Such phenomena is of crucial interest in many engineering applications and are known as coupled thermoelastic problems. This study conducts a variational approach to a numerical simulation that shows a temperature shifting under mechanical loading and deformation of material due to thermal load. Here we consider, in particular, the thermoelastic problem proposed by Biot in [1]:

$$\begin{cases} -\mathrm{div}\ \sigma^*[u,\Theta] &= f(x,t) & \text{in } \Omega \\ c\frac{\partial}{\partial t}\Theta(x,t) &= \kappa\Delta\Theta(x,t) - \Theta_0\beta\frac{\partial}{\partial t}\mathrm{div}(u) + \tilde{f}(x,t) & \text{in } \Omega \\ u &= g & \text{on } \Gamma_D \\ \sigma^*[u,\Theta]n &= q & \text{on } \Gamma_N \\ \Theta &= \tilde{g} & \text{on } \tilde{\Gamma}_D \\ \kappa\frac{\partial\Theta}{\partial n} &= \tilde{q} & \text{on } \tilde{\Gamma}_N \end{cases} .$$

We perform our numerical experiments using the finite element software FreeFEM++ [2] with P1 elements and apply a semi-implicit scheme to solve the given system of partial differential equations. Our results corroborate the fact that mechanical loading can cause a temperature change in a material and that an object may deform in shape due to thermal stress.

Keywords: Thermoelasticity, Finite Element Method, Variational Approach.

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VARIATIONAL APPROACH ON CRACK PATH SELECTION PROBLEM

Alifian Mahardhika Maulana and Masato Kimura

Kanazawa University, Kanazawa, Japan

There are infinite number of possibility of crack path in a material as shown in Figure below. Moreover, mathematical theory about finding crack path is not so established in present. In this occasion, we would like to tackle such problem of finding crack path numerically in variational fracture framework.

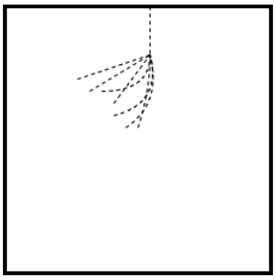

Figure: Crack path on a concrete

In this study, we consider crack propagation problem in a two dimensional isotropic elastic domain. Using the variational crack propagation model by Francfort and Marigo, we investigate straight, kink, and circle crack path then calculate it's energy by velocity method.

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ON ERROR ESTIMATES FOR APPROXIMATE SOLUTIONS OF BIHARMONIC OBSTACLE PROBLEM

Darya Apushkinskaya

Peoples' Friendship University of Russia (RUDN University), Moscow, Russia
Saarland University, Saarbrücken, Germany

In this talk we discuss the bounds of the difference between the exact solution of the variational problem, associated with a free boundary obstacle problem for the biharmonic operator, and any function (approximation) from the energy class satisfying the prescribed boundary conditions and the restrictions stipulated by the obstacle.

Using the general theory developed for a wide class of convex variational problems we deduce the error identity. One part of this identity characterizes the deviation of the function (approximation) from the exact solution, whereas the other is a fully computed value (it depends only on the data of the problem and known functions). In real life computations, this identity can be used to control the accuracy of approximate solutions.

The measure of deviation from the exact solution used in the error identity contains four terms of different nature. Two of them are the norms of the difference between the exact solutions (of the direct and dual variational problems) and corresponding approximations. Two others are not representable as norms. These are nonlinear measures vanishing if the coincidence set defined by means of an approximate solution satisfies certain conditions (for example, coincides with the exact coincidence set).

The error identity is true for any admissible (conforming) approximations of the direct variable, but it imposes some restrictions on the dual variable. We show that these restrictions can be removed, but in this case the identity is replaced by an inequality. For any approximations of the direct and dual variational problems, the latter gives an explicitly computable majorant of the deviation from the exact solution.

The talk is based on results of [1] obtained in collaboration with Sergey I. Repin.

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HIERARCHY OF HYPOPLASTIC MATERIAL MODELS

Erich Bauer

Institute of Applied Mechanics, Graz University of Technology, 8010 Graz, Austria

Based on the concept of hypoplasticity of the Kolymbas type particular constitutive equations relevant to modelling inelastic material properties of frictional granular materials such as sand, gravel and rockfills are considered. In hypoplasticity the constitutive equation is of the rate type and incrementally non-linear. Thus, it allows the modelling of irreversible deformations. In contrast to the concept of elasto-plasticity a decomposition of the deformation into elastic and plastic parts and the introduction of a potential function and flow rule are not needed. These features allow an easy formulation of calibration equations for the material parameters involved in the constitutive equation. Hypoplastic material descriptions are based on terms chosen from the general representation theorem of isotropic tensor valued function of two second order tensors. In the simplest version the evolution equation of the objective stress rate is a function of only the current stress and rate of deformation. Enhanced versions include additional state quantities to take into account the influence of the pressure level, the packing density of the grains and the history of cyclic loading on the incremental stiffness. The predictions of shear strain localization and fluidisation of water saturated granular materials under undrained cyclic shearing are in good agreement with experimental data. Thus, hypoplastic material models are also of interest for the simulation of earthquake phenomena in granular soils. The present paper gives an overview of the hierarchy of hypoplastic constitutive models and demonstrates the performance and limits of particular versions by comparing numerical simulations with laboratory data.

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ON AN EQUILIBRIUM PROBLEM FOR A TWO-LAYER STRUCTURE WITH A CRACK CROSSING THE EXTERNAL BOUNDARY AT ZERO ANGLE

Irina Fankina

Lavrentyev Institute of Hydrodynamics of SB RAS, Novosibirsk, Russia

An equilibrium problem of a two-layer elastic structure with a crack is investigated. The behavior of the layers is modeled within the framework of the two-dimensional elasticity theory. The upper layer is supposed to be glued to the lower one along a part of the edge. Along the gluing line in the lower layer, there is a crack crossing the external boundary at zero angle. On the crack faces, the nonlinear boundary conditions are imposed that exclude their mutual penetration.

Due to the irregularity of the boundary, the possibility of using the fictitious domain method to study the properties of the problem solution was established. The method is based on the introduction into consideration of a family of auxiliary equilibrium problems with a parameter, which are formulated in an extended domain with a smoother boundary. Using the fictitious domain method, the solvability of the problem is proved and the formulations of the limiting equilibrium problems are obtained as the rigidity parameter of the upper layer tends to zero and to infinity.

AN EMPIRICAL PDE FOR SLIP ALONG EARTHQUAKE FAULTS

Shiro Hirano

Dept. of Physical Science, Ritsumeikan Univ., Kusatsu, Shiga, Japan

Dynamic processes of earthquakes have been modelled as spatio-temporal evolution of slip and rupture along shear cracks called as faults. In the simplest case, for example, let the x_1-x_2 plane be a potential fault where slip and rupture propagation occur, and x_1 is the only one direction where slip occurs. Thus, the slip D on the fault is defined as $D(x_1, x_2, t) := \lim_{\varepsilon \to +0} [u_1(x_1, x_2, x_3, t)]_{x_3=-\varepsilon}^{x_3=+\varepsilon}$. Via seismic inversion analyses, seismologists have empirically modelled some power spectra of slip D [e.g., Mai & Beroza, 2002] and slip rate $V := \partial_t D$ [e.g., Kanamori, 2014]. After non-dimensionalization, we found that the empirical models can be summarized as follows:

$$\lim_{k \to 0} \left| \widehat{V}(k, \omega) \right|^2 \sim \left| \frac{\widehat{\epsilon}(\omega)}{1 + 2i\eta\omega - \omega^2} \right|^2, \tag{1}$$

$$\lim_{\omega \to 0} \left| \widehat{V}(k, \omega) \right|^2 \sim \left| \frac{\widehat{\epsilon}(k)}{1 + k^2} \right|^2, \tag{2}$$

where $\widehat{\epsilon}$ has almost constant power spectrum (i.e., $|\widehat{\epsilon}(\omega)|^2 \sim 1$, and $|\widehat{\epsilon}(k)|^2 \sim 1$) with some fluctuation, and

$$\widehat{V}(k, \omega) := \frac{1}{2\pi} \int_{\theta=-\pi}^{+\pi} \int_{r=0}^{\infty} \int_{t=0}^{\infty} V(r, \theta, t) e^{i\omega t - ikr\cos\theta} r \, dt \, dr \, d\theta \tag{3}$$

is the curcular average of spatio-temporal Fourier transform with $r = \sqrt{x_1^2 + x_2^2}$, $\theta = \arctan \frac{x_2}{x_1}$, and k and ω are absolute value of the wavenumber vector and angular frequency, respectively.

Although there is no unique spectrum $\widehat{V}(k, \omega)$ that satisfies both (1) and (2), we propose a simple model equation as follows:

$$(\partial_t^2 + 2\eta\partial_t - \Delta + 1) V(x_1, x_2, t) = \epsilon(x_1, x_2, t). \tag{4}$$

Eq.(4) includes some reasonable properties in comparison to the results from seismic inversion analyses: i) slip-rate V shows the empirical power spectrum (1) and (2), ii) V evolves just like wave front ($\because \partial_t^2 - \Delta$), iii) V dissapears within a finite period ($\because +2\eta\partial_t$), and iv) the final state $\int_{\mathbb{R}^2} dx_1 dx_2 \int_0^\infty dt V(x_1, x_2, t) = \widehat{V}(0, 0)$ has non-zero and finite value (because of the mass term). We discuss some similarities between the solution of eq.(4) and observational properties of earthquake faulting processes.

The work was funded by JSPS the Grants-in-Aid for Scientific Research (KAKENHI) Program No. 17H02857 and No. 18K13637.

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ON A FLAT-PUNCH INDENTATION PROBLEM WITHIN THE CONTEXT OF LINEARIZED VISCOELASTICITY

Hiromichi Itou

Department of Mathematics, Tokyo University of Science, Tokyo, Japan

In this talk, we deal with the indentation of a flat-ended cylindrical rigid punch into a viscoelastic half-space [4]. This is closely related to the Boussinesq problem [1] for finding the deformation in the case of a concentrated load applied on the plane boundary, by passing to the limit as the punch radius tends to zero. For this problem, we adopt a linear viscoelastic model wherein the linearized strain is expressed as a function of the stress, admitting the case it is not invertible. It is well known that solution of problems in linear viscoelasticity can be obtained by virtue of a correspondence principle between the solutions in linearized elasticity and linear viscoelasticity. However, for this principle to be applicable certain conditions with regard to the deformation and the loading have to be satisfied, refer [5] for the details. The problem under consideration is an example where one cannot appeal to the correspondence principle. Moreover, it is necessary to remark that while the solution that we seek are time dependent due to the material under consideration being viscoelastic, we are not considering inertial effects. Thus, we are only interested in solving the balance of linear momentum in the absence of inertial effects.

Then, based on the Papkovich-Neuber representation in potential theory (cf. [2]) and use of the Fourier-Bessel transform for axisymmetric bodies, an analytical solution of the resulting time-dependent integral equation is constructed. As the result, distribution of the displacement and the stress fields in the half space with respect to time is obtained in the closed form. The existence of the solution of the problem is discussed in [3] for more general situations.

This work is based on a joint project with V. A. Kovtunenko(Univ. of Graz/Lavrentyev Inst. of Hydrodynamics) and K. R. Rajagopal(Texas A&M Univ.). The work was funded by JSPS and RFBR under the Japan - Russia Research Cooperative Program No. J20-720, also partially supported by Grant-in-Aid for Scientific Research (C)(No. 18K03380) and (B)(No. 17H02857) of Japan Society for the Promotion of Science.

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UNIQUE SOLVABILITY OF A CRACK PROBLEM WITH SIGNORINI-TYPE AND GIVEN-FRICTION CONDITIONS IN A LINEARIZED ELASTODYNAMIC BODY

Takahito Kashiwabara

The University of Tokyo, Tokyo, Japan

Let $\Omega \subset \mathbb{R}^d\,(d = 2, 3)$ be a bounded smooth domain, whose boundary consists of disjoint parts Γ_D and Γ_N. Moreover, Ω is separated into two Lipschitz domains $\Omega_\pm$ by an interface $\Gamma = \partial\Omega_+ \cap \partial\Omega_-$. We further assume that there is a crack in Ω, which is a subset of Γ and denoted by Γ_c. Let n represent the outer unit normal on $\partial\Omega$ and the unit normal on Γ directing from Ω_- to Ω_+. Then the jump quantity across Γ is written as $\cdot = \cdot|_{\Gamma_+} - \cdot|_{\Gamma_-}$. We finally define $\Omega_c := \Omega \setminus \overline{\Gamma}_c$ (such a setting can be found in [3]).

For a displacement field u, we assume that the linear elastic tensor is given as $\sigma = \lambda \operatorname{div} u\, \mathbb{I} + \mu\mathbb{E}(u)$, where $\mathbb{E}(u) = \nabla u + (\nabla u)^\top$ and $\lambda, \mu > 0$. We consider the following non-stationary linear elasticity equations with a Signorini-type condition and given-friction condition (also known as Tresca condition) imposed on the crack Γ_c:

$$
\begin{array}{rll}
u'' - \operatorname{div}\sigma = f & \text{in} & (0,T)\times\Omega_c,\\
u = 0 & \text{on} & (0,T)\times\Gamma_D,\\
\sigma n = F & \text{on} & (0,T)\times\Gamma_N,\\
(\sigma n)_n = 0,\ (\sigma n)_n \le 0,\ \delta u_n + u'_n \ge 0,\ (\sigma n)_n \delta u_n + u'_n = 0 & \text{on} & (0,T)\times\Gamma_c,\\
(\sigma n)_\tau = 0,\ |(\sigma n)_\tau| \le g,\ (\sigma n)_\tau \cdot u'_\tau = g|u'_\tau| & \text{on} & (0,T)\times\Gamma_c,\\
u(0) = u_0, \quad u'(0) = \dot{u}_0 & \text{in} & \Omega_c,
\end{array}
$$

where $\delta \in [0, \infty)$ is a constant, and the subscripts n and τ mean the normal and tangential components of vectors, respectively. The data are assumed to satisfy $f \in H^1(0,T;H)$, $F \in H^2(0,T;L^2(\Gamma_N)^d)$, $g \in H^2(0,T;L^2(\Gamma_c))$, and $u_0, \dot{u}_0 \in V$, where $H = L^2(\Omega_c)^d$ and $V = \{v \in H^1(\Omega_c)^d \mid v = 0 \text{ on } \Gamma_D\}$. Then our main result is stated below.

Theorem. *Under some compatibility conditions there exists a unique solution* $u \in W^{2,\infty}(0,T;H) \cap W^{1,\infty}(0,T;V)$ *of the above problem. In particular, for a.e.* $t \in (0,T)$ *one has* $\delta u(t) + u'(t) \in K = \{v \in V \mid v_n \ge 0 \text{ on } \Gamma_c\}$ *and a hyperbolic variational inequality*

$$
\begin{aligned}
&(u''(t), v - (\delta u(t) + u'(t))) + a(u, v - (\delta u(t) + u'(t))) + (g(t), |v_\tau - \delta u_\tau(t)| - |u'_\tau(t)|)_{\Gamma_c}\\
&\ge (f(t), v - (\delta u(t) + u'(t))) + (F(t), v - (\delta u(t) + u'(t)))_{\Gamma_N} \qquad \forall v \in K,
\end{aligned}
$$

where $(\cdot,\cdot) = (\cdot,\cdot)_{L^2(\Omega_c)}$, $(\cdot,\cdot)_{\Gamma_c} = (\cdot,\cdot)_{L^2(\Gamma_c)}$, $a(u,v) = \lambda(\operatorname{div} u, \operatorname{div} v) + \frac{\mu}{2}(\mathbb{E}(u), \mathbb{E}(v))$.

The novelty of this result can be explained as follows. First, we consider a dynamic elasticity problem with friction, in which the threshold g may depend on time; compare with [1, p. 159] which claimed that $\partial_t g$ should be 0. Second, we generalized the contact condition imposed on velocity formulated e.g. in [2, p. 264], noting that $\delta = \infty$ formally corresponds to the usual contact condition imposed on displacement. When $\delta \neq 0$, the a priori estimates and uniqueness proof become non-trivial. We also emphasize that u'' exists as a usual function in $L^\infty(0, T; L^2(\Omega_c)^d)$ rather than a distribution.

This is a joint work with Hiromichi Itou.

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EQUILIBRIUM PROBLEM FOR ELASTIC BODY WITH DELAMINATED T-SHAPE INCLUSION

Alexander Khludnev

Lavrentyev Institute of Hydrodynamics of SB RAS,
Novosibirsk State University, Novosibirsk, Russia

The talk concerns an equilibrium problem for 2D elastic body with a T-shape thin inclusion in presence of damage. A part of the inclusion is elastic, and the other part is a rigid one. A delamination of the inclusion from the elastic body is assumed, thus forming a crack between the elastic body and the inclusion. Nonlinear boundary conditions at the crack faces are considered to prevent a mutual penetration between the faces. The damage is characterized by a positive parameter. The paper provides an asymptotic analysis of the solutions as the damage parameter tends to infinity and to zero. A passage to infinity of a rigidity parameter of the elastic part of the inclusion is also analyzed. Junction conditions are determined at the connection point between the elastic and rigid parts of the inclusion. An existence theorem is proved for an inverse problem of finding displacement fields and the damage and rigidity parameters provided that a displacement of the tip point of the inclusion is known.

The work was funded by the Russian Foundation for Basic Research according to the project No. 19-51-50004.

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A UNIFIED MODEL FOR STRESS-DRIVEN REARRANGEMENT INSTABILITIES

Shokhrukh Kholmatov

University of Vienna, Vienna, Austria

In this talk I will speak about a variational model to simultaneously treat Stress-Driven Rearrangement Instabilities (SDRI) introduced in [1, 2], such as boundary discontinuities, internal cracks, external filaments, edge delamination, wetting, and brittle fractures, is introduced. The model is characterized by an energy displaying both elastic and surface terms, and allows for a unified treatment of a wide range of settings, from epitaxially-strained thin films to crystalline cavities, and from capillarity problems to fracture models.

Existence of minimizing configurations is established in two-dimensions by adopting the direct method of the Calculus of Variations. Compactness of energy-equibounded sequences and energy lower semicontinuity are first shown with respect to a proper selected topology in a class of (constraint) admissible configurations under the assumption that the free crystalline interface is the boundary, consisting of an at most fixed finite number m of connected components, of sets of finite perimeter.

Next we show that, as $m \to \infty$, the energy of constraint minimal admissible configurations tends to the minimum energy in the general class of configurations without the bound on the number of connected components for the free interface. Also by means of those constraint m-minimizers as well as uniform density estimates for the local decay of the energy at the m-minimizers' boundaries, we will directly construct the global (unconstraint) minimizing candidate of the SDRI model. Finally, we study some regularity properties for the morphology of any minimizer.

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CONVERGENCE OF (ADAPTIVE) APPROXIMATION SCHEMES FOR RATE-INDEPENDENT SYSTEMS

Dorothee Knees

Institute for Mathematics, University of Kassel, Kassel, Germany

It is well known that rate-independent systems involving nonconvex stored energy functionals in general do not allow for time-continuous solutions even if the given data is smooth in time. Several solution concepts are proposed to deal with these discontinuities, among them the meanwhile classical global energetic approach and the more recent vanishing viscosity approach. Both approaches generate solutions with a well characterized jump behavior. However, the solution concepts are not equivalent. In this context, numerical discretization schemes are needed that efficiently and reliably approximate directly that type of solution that one is interested in. For instance, in the vanishing viscosity context it is reasonable to couple the viscosity parameter with the time-step size. However, the numerical examples from [1] show that even knowing the exact solution it is extremely difficult to choose viscosity and time-discretization parameters in such a way that the correct jump behavior is visible already for rather coarse discretizations. The aim of this lecture is to discuss different time-discretization schemes, to study their convergence and to characterize as detailed as possible the limit curves as the discretization parameters tend to zero. The main focus will lie on alternate minimization schemes that are quite popular in the context of damage models. Switching to a time-reparametrized picture, the behavior at jump points can be made visible and similarities and differences to other approaches will be discussed. The part on alternate minimization schemes is joint work with M. Negri, Pavia, [2].

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ON SOLUTION OF INITIAL BOUNDARY VALUE PROBLEMS IN HYPOPLASTICITY

Victor Kovtunenko

Institute for Mathematics and Scientific Computing, Karl-Franzens University of Graz, NAWI Graz, Austria
Lavrentyev Institute of Hydrodynamics of SB RAS, Novosibirsk, Russia

The hypoplasticity is motivated by engineering application to granular materials and soils in geomechanics. The reference hypoplastic constitutive equations of Kolymbas type are fully nonlinear and rate-independent, they are a practice example of implicit theories as suggested by Truesdell and Rajagopal. Mathematically, our study is faced to ill-posed dynamic PDE problems. Moreover, for a cohesionless granular material the constraint of non-positive principal stresses preserving compression of grains should be satisfied within the solution. The well-posedness analysis results known in the literature are restricted for hypoplastic models simplified either to a semi-discrete nonlinear Cauchy problem following Chambon, or to quasi-static nonlinear rate problems. For the particular models of Bauer, von Wolffersdorff, Toll, and others, we aim at the dynamic behavior of the nonlinear ODE systems under proportional and cyclic loading, which leads to hysteresis and ratcheting phenomena, especially important for applications.

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A MODEL FOR PHASE TRANSITIONS IN ELASTOPLASTIC POROUS MEDIA

Pavel Krejčí

Faculty of Civil Engineering, Czech Technical University, Prague
Institute of Mathematics, Czech Academy of Sciences, Prague

We propose and study a continuum model for fluid diffusion in a deformable elastoplastic porous medium, where the fluid may undergo phase transitions. Typically, such problems arise in modeling liquid-solid phase transformations in groundwater flows. The system of equations is derived here from the conservation principles for mass, momentum, and energy and from the Clausius-Duhem inequality for entropy. It couples the evolution of the displacement in the matrix material, of the capillary pressure, of the absolute temperature, and of the phase fraction. For the resulting nonlinear system of three PDEs (mass balance, momentum balance, energy balance) and one ordinary differential inclusion related to the relaxed Stefan problem for the evolution of the phase fraction. The system is coupled with natural initial and boundary conditions and under suitable assumptions on the constitutive laws, we prove the existence of global in time solutions. In the isothermal case, we prove in [3] that the system is asymptotically stable as time tends to infinity.

The presence of two hysteresis operators, namely in the pressure-saturation relation in the fluid and in the stress-strain relation in the elastoplastic solid, makes the problem challenging and only partial results have been published so far in [1, 2, 5]. The full system has been solved only recently by Chiara Gavioli in her PhD Thesis [4].

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INFLUENCE OF TECHNOLOGY OF HOT FORMING OF PLATES FROM ALUMINUM ALLOYS V-1461 (AL-CU-LI-ZN) AND V95 (AL-ZN-MG-CU) ON RESISTANCE TO FATIGUE FRACTURE

Alexey Larichkin and Kirill Zakharchenko

Lavrentyev Institute of Hydrodynamics of SB RAS, Novosibirsk, Russia

The work is devoted to assessing the effect of creep pressure treatment of an aluminum alloy V-1461 (Al-Cu-Li-Zn) on fatigue fracture resistance. With the help of the developed accelerated, non-destructive method, the kinetics of the response of total deformations (longitudinal and transverse components) is considered when the sample is deformed by an increasing load. Within the framework of this method, the limiting stresses are determined, a conservative estimate of the endurance limit for the aluminum alloy V-1461 (analogue 2099) is obtained. The shaping of thick plates (40 mm) was carried out on an UFP-1M installation (NAZ named after V.P. Chkalov) in the creep mode and surface control.

More than 80% of the slab area is molded with a deviation of less than 1 mm from the nominal size. The microstructure of the V-1461 alloy before and after shaping was investigated, and fatigue tests were carried out on specimens made of molded panels. Analysis of the results of testing materials showed that for alloys V95 and V-1461 the selected characteristics of the technological process of shaping and heat treatment do not worsen the fatigue properties of the studied alloys.

Mathematical modeling of deformation of plates under creep conditions was carried out and a comparison with experimental data for materials V95 and V-1461 was carried out. Modeling has shown that the law of steady-state creep is not sufficient to describe the process; the need to formulate the inverse problem of shaping is noted, where the location and movement of the form punches relative to the slab are the boundary conditions.

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EQUILIBRIUM PROBLEM FOR AN THERMOELASTIC KIRCHHOFF–LOVE PLATE WITH A DELAMINATED RIGID INCLUSION

Nyurgun Lazarev

North-Eastern Federal University, Yakutsk, Russian Federation

We formulate a new variational problem on the equilibrium of a thermoelastic Kirchhoff–Love plate in a domain with a cut. We suppose that the plate has a vertical crack at the boundary of a volume rigid inclusion. It is assumed that the plate is under the special loads for which the configuration of crack's edges is known in advance. This circumstance leads to new special relations describing the possible mechanical interaction of opposite crack edges. The initial formulation of a problem presupposes the fulfillment of boundary conditions on the crack curve in the form of system of two inequalities and an equality. Using the approach developed in the work [1], the solvability of the problem is proved, an equivalent differential setting is found.

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SHAPE OPTIMIZATION OF SINGULAR POINTS IN BOUNDARY VALUE PROBLEMS OF PARTIAL DIFFERENTIAL EQUATIONS

Kohji Ohtsuka

Hiroshima Kokusai Gakuin University, 6-20-1, Nakano Aki-ku, Hiroshima Japan

The principal objective of this work is the systematic development of the general theory of shape optimization of singular points of a weak solution in boundary value problem (BVP) of partial differential equations (PDEs) [5, 8] including fracture mechanics, shape optimization of boundary and interface [4] etc. This means we shall be concerned with the shape sensitivity optimization and numerical analysis of singular points, that is, the points on boundary, cracks and Interface created by PDEs with discontinuous coefficients, using GJ-integral. Here, the GJ integral was proposed by the author in 1981 [1] to represent the energy release rate in a three-dimensional fracture phenomenon. After that, it was enlarged so that it could be applied to the boundary shape sensitivity analysis of energy cost functionals in elliptic PDE boundary value problem [2], and its effectiveness in the mixed boundary value problem was shown [7]. Also, GJ-integral can use to express the shape sensitivity of interface created by PDEs with discontinuous coefficients. Using GJ-integral and adjoint variable method, we can derive weak formulation of Hadamard variation applicable to various BVP [3], and using it to lead shape sensitivity of least mean-squre error, etc.By using the H1-gradient method and GJ-integral, it is possible to find the shape optimization of boundaries (with mixed boundary) and interfaces. Others include GJ-integral, which can be applied to eigenvalue problems, and problems that use the method of Lagrange multipliers [6].

In my talk, I will show how to use the GJ-integral for Poisson's equation in two-dimensional case, but similarly, method in my talk can be applied to systems such as elasticity and nonlinear problems regardless of dimension.

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APPROXIMATIONS OF RESOLVENT IN HOMOGENIZATION OF FOURTH ORDER ELLIPTIC OPERATORS

Svetlana Pastukhova

MIREA – Russian Technological University, Moscow, Russia

Consider divergence-type fourth order elliptic operators

$$A_\varepsilon = \sum_{i,j,s,t} D_{ij} a_{ijst}(x/\varepsilon) D_{st}, \quad D_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad x \in \mathbb{R}^d (d \geq 2), \quad \varepsilon \in (0,1), \tag{1}$$

with ε-periodic coefficients. Assume that the fourth order tensor $a(x)=\{a_{ijst}(x)\}$, $1 \leq i,j,s,t \leq d$, is periodic ($Y=[0,1)^d$ is a cell of periodicity), real, bounded measurable and satisfies the following conditions of symmetry and ellipticity: (i) $a_{ijst}=a_{stij}$, $a_{ijst}=a_{jist}=a_{ijts}$; (ii) $\lambda\xi \cdot \xi \leq a(\cdot)\xi \cdot \xi \leq \lambda^{-1}\xi \cdot \xi$ for any symmetric matrix $\xi=\{\xi_{ij}\}$, where $\lambda > 0$. The homogenized operator $\hat{A}$ is of the same kind, $\hat{A} = \sum_{i,j,s,t} D_{ij} \hat{a}_{ijst} D_{st}$, but with constant coefficients which are defined via solutions to auxiliary problems on the cell of periodicity Y. Operators of the type (1) appear in the study of elastic thin plates made of composite materials with periodic structure. The effective description for these heterogeneous media, as ε, the size of periodicity cell, tends to zero, is most in demand for applications.

We are interested in approximations for the resolvent $(A_\varepsilon + 1)^{-1}$ in operator norms and find corresponding error estimates over the small parameter ε. In [1],[2], the approximation $(A_\varepsilon + 1)^{-1} = (\hat{A}+1)^{-1} + \varepsilon^2 K(\varepsilon) + O(\varepsilon)$ was obtained in the "energy" norm (i.e., in $(L^2 \to H^2)$-norm), where, as a simple corollary, the corrector term $\varepsilon^2 K_\varepsilon$ may be dropped, due to its properties, if the resolvent approximation of order ε is sought in a weaker $(L^2 \to L^2)$-norm. In [3],[4], it was proved that actually $(A_\varepsilon + 1)^{-1} = (\hat{A} + 1)^{-1} + O(\varepsilon^2)$ in L^2 operator norm; but if we omit the symmetry condition (i) on the tensor $a(x)$, this approximation with remainder term of the order ε^2 should be corrected in the following manner: $(A_\varepsilon + 1)^{-1} = (\hat{A}+1)^{-1} + \varepsilon K_1 + O(\varepsilon^2)$. There are constructive formulae to define the correcting operators $K(\varepsilon)$ and K_1 via the solutions to the aforementioned cell problems. In [3], these results are extended to the case of elliptic divergence-type operators of the arbitrary even order $2m \geq 4$.

In [5], another model of elastic thin plates is considered, which evolves singularly perturbed operators. Here, the effective description, comprising resolvent approximations, is quite different and resembles the case of second order elliptic operators.

To prove our results we use the shift method suggested by V.V. Zhikov in 2005.

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ON JUNCTION PROBLEM FOR TIMOSHENKO AND RIGID THIN INCLUSIONS IN 2D ELASTIC BODY

Tatiana Popova

North-Eastern Federal University, Yakutsk, Russia

The research concerns a junction problem for a thin elastic inclusion contacting with a thin rigid inclusion inside a two-dimensional elastic body. Timoshenko's theory of thin beams is used to describe the model of the elastic inclusion. Various cases of junction of thin inclusions are considered. In the first case, the inclusions are in contact as two separate objects. In another case, inclusions are one whole inclusion, consisting of parts with different physical properties. In this case, the condition for the absence of a break is considered, as well as the problem with the damage parameter characterizing a connection between two inclusions. Both inclusions are of a rectilinear shape and delaminate from the elastic matrix. Therefore, the problem is posed in the domain with a cut and conditions of the form of inequalities are specified at the edges of the crack, as on a part of the boundary. These conditions exclude mutual penetration of the crack edges into each other. At the same time, such a formulation leads to the nonlinearity of the problem and the need to use additional mathematical methods to construct an algorithm for the numerical solution of the problem [1-4]. Differential and variational statements for the cases of delaminated thin inclusions are presented. A junction conditions at the connection point are written out. For the numerical solving of the problem in a domain with a cut, a variational formulation is used using the domain decomposition method and the Uzawa algorithm. Examples of a computational experiment are given.

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A CONTACT OF TWO ELASTIC PLATES EACH CONTAINING A CRACK

Evdokia Pyatkina

Lavrentyev Institute of Hydrodynamics of SB RAS, Novosibirsk, Russia

A contact of two elastic plates located in parallel is considered. Each of the plates containes a crack of a same shape and lenght. The plates are connected to each other along the faces of the cracks. It is assumed that a force acts between the plates both along and perpendicular to the contact planes. This force is proportional to the difference between the displacements of points of an upper surface of the lower plate and a lower surface of the upper plate [1,2]. Non penetration conditions between the plates and at the crack faces are taken into account [3]. A unique solvability of the problem is shown. An asymptotic analysis is performed when the value of the force acting between the plates tends to zero or to the infinity.

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JUSTIFICATION OF MODELS OF PLATES CONTAINING INSIDE HARD THIN INCLUSIONS

Evgeny Rudoy

Lavrentyev Institute of Hydrodynamics of SB RAS, Novosibirsk, Russia
Novosibirsk State University, Novosibirsk, Russia

An equilibrium problem of a Kirchhoff–Love plate containing a nonhomogeneous inclusion is considered. It is assumed that the elastic properties of the inclusion depend on a small parameter characterizing width of the inclusion ε as ε^N with $N < 1$. The problem is formulated as a variational one; namely, as a minimization problem of the energy functional over a set of admissible deflections in the Sobolev space H^2. This implies that the deflections function is a solution of a boundary value problem for bi-harmonic operator (pure bending, see, e.g., [1, 2]).

The aim of the present work is to justify passing to the limit as $\varepsilon \to 0$. To do this, we apply a method that was originally introduced in [3, 4] for problems of gluing plates. The method is based on variational properties of the solution to the corresponding minimization problem and allows for finding a limit problem for any $N < 1$ simultaneously. It is shown that there exist two types of hard inclusions in dependence of N: thin rigid inclusion ($N < -1$) and thin elastic inclusion ($N = -1$). In case $N \in (-1, 1)$, the influence of the inhomogeneity disappears in the limit. We get limit problems in a variational form, which is convenient, for example, for numerical analysis by the finite element method.

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A PENALIZED VERSION OF THE LOCAL MINIMIZATION SCHEME FOR RATE-INDEPENDENT SYSTEMS

Viktor Shcherbakov

Institute of Mathematics, University of Kassel, Kassel, Germany
Lavrentyev Institute of Hydrodynamics of SB RAS, Novosibirsk, Russia

We present a penalized version of the time-discretization local minimization scheme first proposed by Efendiev and Mielke to resolve time discontinuities in rate-independent systems with nonconvex energies. In order to penalize inequality constrains enforcing the local minimality, a Moreau–Yosida approximation is employed. We prove the convergence of time-discrete solutions to functions that are parametrized BV solutions of the time-continuous problem (in an abstract infinite-dimensional setting), provided that the discretization and approximation parameters are chosen appropriately. We test our scheme on a one-dimensional example and find a notable improvement compared with the original version.

This is a joint work with Dorothee Knees.

INVERSE PROBLEMS FOR GENERAL FIRST-ORDER HYPERBOLIC EQUATIONS

Hiroshi Takase

Graduate School of Mathematical Sciences, The University of Tokyo, Tokyo, Japan

Let $d \geq 1$ and $\Omega \subset \mathbb{R}^d$ be a bounded domain with smooth boundary $\partial\Omega$. Set $Q := \Omega \times (0, T)$ for $T > 0$. We consider the first-order partial differential operator P such that

$$Pu := A^0(x,t)\partial_t u + A(x,t) \cdot \nabla u,$$

where $A^0 \in C^1(\overline{Q})$ is a positive function and $A = (A^1, \dots, A^d) \in C^2(\overline{Q}; \mathbb{R}^d)$ is a vector-valued function. Define an outgoing boundary $\Sigma_+ := \{(x,t) \in \partial\Omega \times (0,T) \mid A(x,t) \cdot \nu(x) > 0\}$ and incoming boundary $\Sigma_- := \big(\partial\Omega \times (0,T)\big) \setminus \Sigma_+$, where ν denotes the outer unit normal to $\partial\Omega$. In this talk, we consider the following two kinds of inverse problems.

Inverse source problem

Let u be a function of a class satisfying

$$\begin{cases} Pu + p(x,t)u = R(x,t)f(x) & \text{in } Q, \\ u = 0 & \text{on } \Sigma_-, \\ u(\cdot,0) = 0 & \text{on } \Omega, \end{cases}$$

where $p \in W^{1,\infty}(0,T;L^\infty(\Omega))$, $R \in H^1(0,T;L^\infty(\Omega))$, and $f \in L^2(\Omega)$. We will see global Lipschitz stability to reconstruct f in Ω from observation data u on Σ_+.

Inverse coefficient problem

For $m = 1,\dots,d+1$, let u_m be a function of a class satisfying

$$\begin{cases} Pu_m + p(x,t)u_m = 0 & \text{in } Q, \\ u_m = g_m & \text{on } \Sigma_-, \\ u_m(\cdot,0) = \alpha_m & \text{on } \Omega, \end{cases}$$

where $p \in W^{1,\infty}(0,T;L^\infty(\Omega))$, $g_m \in L^2(\Sigma_-)$, and $\alpha_m \in L^2(\Omega)$. We will see global Lipschitz stability to reconstruct $\{A^\mu\}_{\mu=0}^d$ from finitely many observation data $\{u_m\}_{m=1}^{d+1}$ on Σ_+.

This is a joint work with Professor Giuseppe Floridia (Mediterranea University of Reggio Calabria, Italy).

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INVERSE SCATTERING PROBLEMS ON QUANTUM GRAPHS

Igor Trushin

Shinshu University, Matsumoto, Japan

I investigate inverse scattering problems for a Sturm-Liouvile operator on the quantum graphs. The scattering matrix, part of the negative eigenvalues and corresponding normalizing coefficients are taken as a scattering data. The main goal of this research is to reconstruct the coefficients of Sturm-Liouvile operator on the basis of the given scattering data. I have deduced Marchenko equation which allowed us to prove the uniqueness theorems and provided a reconstruction procedure for the coefficients on the half-lines.

Most of the results of this presentation were obtained jointly with Prof. K.Mochizuki.