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RUSSIA-JAPAN WORKSHOP

MATHEMATICAL ANALYSIS OF FRACTURE PHENOMENA FOR ELASTIC STRUCTURES AND ITS APPLICATIONS

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ABSTRACTS

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The mathematical foundation of fracture mechanics has seen considerable advances in the last years. This field of study covers a big variety of exciting topics, including propagation of cracks, equilibrium of structures with thin inclusions in the presence of delaminations, frictional contact problems, inverse and control problems. The aim of the workshop "Mathematical analysis of fracture phenomena for elastic structures and its applications" is to bring together researchers working on different aspects of these issues. The workshop provides a platform for researchers from Russia and Japan to communicate, discuss, and exchange ideas under the common theme of fracture phenomena.

The workshop topics:

- crack propagation
- modeling of composite materials
- identification of defects in elastic bodies
- study of mathematical models for solids with defects
- investigation of the asymptotic behavior of solutions

The workshop is supported by Japan Society for the Promotion of Science, Russian Foundation for Basic Research under the Japan–Russia Research Cooperative Program, and Novosibirsk State University.

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Contents

NUMERICAL SIMULATION OF DEFORMATION AND FRACTURE OF CONCRETE AND FIBER CONCRETE STRUCTURES WITH METAL AND NON-METAL REINFORCEMENT UNDER PULSE LOADING

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Manufacturing durable and high-strength concrete structures has always been a relevant objective. While designing new structures along with analyzing behavior of separate elements under different impacts the overall structural behavior should be also analyzed. Full- scale experiments in this case would generally imply large material costs though the required result will not always be obtained. This significantly concerns dynamic processes requiring information on particular parameters at different periods. Thus, the need arises in models and methods enabling to perform analysis and predict structural behavior under different operational loads and possible abnormal situations. Proper definition of structural behavior should take into account 3D nature of the stress-strain state specified by several factors: presence of elements leading to geometrical asymmetry; consideration of the real loading conditions generally being non-symmetrical; anisotropy of physical-mechanical properties of materials that structural elements are made of.

In case one of the given factors takes place, it defines carrying 3D analysis. This complex task implicates developing adequate model describing structural behavior as well as real consideration of geometry and spatial configuration of different elements of structure.

The research presents the following results:

- results of integrated experimental and numerical study of destruction of reinforced concrete beam made of concrete and fiber concrete under short-term dynamic loading. Experimental studies were carried out using pile driver. Short-term dynamic loading acting on a beam was applied by falling weight, 450 kg, from the height 700 mm;

- results of numerical simulation of interaction between aircraft Boeing 747-400 and protective shell of nuclear power plant. The shell is presented as complex multilayered cellular structure comprising layers of concrete and fiber concrete bonded with steel trusses;

- results of experimental and numerical investigation of nature of destruction and crack formation in concrete beams with rod composite reinforcement. Concrete elements have been tested under dynamic load using special pile driver. The obtained results are: patterns of destruction and crack formation, maximum load value and maximum element deflection.

Numerical simulation was held three-dimensionally using the authors algorithm and software taking into account algorithms for building grids of complex geometric objects and parallel computations. The dynamics of stress-strain state and fracture of structure were studied. Destruction is described using two stage model that allows taking into account anisotropy of elastic and strength properties of concrete and fiber concrete.

The work was funded by the Russian Foundation for Basic Research according to the projects № 18-41-703003 and № 18-48-700035.

HYDRAULIC FRACTURING OF PERFORATED CONCRETE CYLINDERS IN A NON-UNIFORM STRESS FIELD

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The technology of hydraulic fracturing is widely used in oil and gas production. However, the analysis of fracture under inhomogeneous stress state around the borehole is problematic. In this study, we compared numerical calculations for different fracture criteria with experimental data on hydraulic fracturing of cylindrical specimens made of concrete in an inhomogeneous stress field, in order to determine the most suitable for this purpose. Real wells have a significantly larger diameter than laboratory models. Therefore, it is important to investigate the scale factor, which, according to the literature, takes place during hydraulic fracturing. It is known that in the presence of stress concentration, the scale factor manifests itself at a much greater degree than under a homogenous stress state. An experimental study of the hydraulic fracture of thick-walled cylinders with the application of a diametrical load was carried out on specimens in the form of cylinders with a central circular hole and a circular hole located at a distance of half the radius from the center of the cylinder at an angle of 45 degrees to the load application line. Experiments on hydraulic fracturing were carried out using the machine that creates a high oil pressure. Cylinders of different diameters with different hole diameters were tested to study the scale effect. Specimens were made of concrete based on aluminous cement. The preparation of a solution for casting samples was carried out by mixing sand, cement and water in the proportion 3/2/1. The value of the critical stress intensity factor of concrete that was used in non-local fracture criteria was obtained by compression of the cylindrical specimens with a notch. The modeling of the fracturing process taking into account the inhomogeneity of the stress state near the hole was carried out using the boundary elements method (in the variant of the fictitious load method), the local maximum stress criterion and the gradient fracture criterion [1]. In samples with a central hole, the hydraulic fracture cracks propagated from the hole contour along the load application line. In samples with a hole offset from the load application line, fracture cracks spread from the hole contour in the direction of the linear contact area of the cylinder with the loading system and in the opposite direction. It is concluded that the local maximum stress criterion gives significantly lower values of the breakdown pressure during hydraulic fracturing. It is shown that using the gradient criterion, it is possible to obtain a satisfactory agreement between the results of calculations and experimental data on hydraulic fracturing in non-uniform stress fields. Experimental study showed scale effect in concrete specimens with stress concentrators.

The work was partially supported by the Government of the Russian Federation (Grant No. 14.W03.31.0002) and by the Russian Foundation for Basic Research (project 18-08-00528).

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ON AN EQUILIBRIUM PROBLEM FOR A TWO-LAYER STRUCTURE WITH THE UPPER LAYER COVERING A DEFECT TIP

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In the framework of the plane problem in elasticity theory, the equilibrium problem of a two-layer elastic structure is investigated. In the lower layer of the structure there is a rectilinear defect. The upper layer covers one of the defect tips and is glued to the lower layer along its edge. Nonlinear boundary conditions are used to describe the defect, which allow avoiding the phenomenon of mutual penetration of the opposite defect faces. In addition, the boundary conditions contain a damage parameter characterizing the defect: the greater its value, the weaker the adhesion of the defect faces.

Using the variational approach, the solvability of the problem is established. Passage to the limit in the problem with respect to the rigidity parameter of the upper layer is carried out. We also study the behavior of the problem solution when the defect damage parameter tends to zero and to infinity. In addition, the optimal control problem formulated on the basis of the Griffith's criterion is considered. The cost functional in the problem is the derivative of the energy functional with respect to the defect length. Two parameters mentioned above are chosen as control functions. The existence of a solution to the optimal control problem is proved.

The work was funded by the Russian Foundation for Basic Research according to the project № 18-29-10007.

ON UNILATERAL CONTACT BETWEEN A TIMOSHENKO PLATE AND A THIN ELASTIC OBSTACLE

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In this talk, several problems about unilateral contact between a plate and a thin obstacle will be presented. Within the framework of model under consideration the plate is described by Timoshenko theory and the thin obstacle is described by Euler–Bernoulli beam theory. Mutual penetration can not occur during contact. A study is focused on the well-posedness of the equilibrium problems as well as the limit transitions and differentiability of the energy functional.

The reported study was funded by the Russian Foundation for Basic Research, project number 19-31-90037.

THE PROBLEM OF THE FORMATION OF ANNULAR CRACKS IN THE GLASS HALF-SPACE UPON CONTACT WITH THE BALL, TAKING INTO ACCOUNT REAL EXPERIMENTAL CONDITIONS

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The contact problem of the ball indentation into the glass half-space is considered. The optical microscope is installed on the lower base of the traverse of the testing machine. The microscope is closed with the steel protection with the round hole for microscope to observe the specimen. The glass specimen in the form of the parallelepiped with the sides of 20 mm and the height of 10 mm is placed on the hole. The steel ball of different diameters (5.5 mm; 10 mm and 17 mm) is pressed into the center of the specimens.The experimental formulation is the contact problem [1]. The maximum tensile stress at the point of crack formation, that is, the radial stress, was interested, since annular cracks in the experiment were formed on the surface in the vicinity of the contact area of the specimen. In addition, it was necessary to determine the effect of the hole for the microscope on the stresses in the vicinity of the contact aria. Huber's solution was used to analytically calculate the radial stress at any point in the half-space [2]. However, this solution does not take into account the bending stresses arising from the hole for the microscope. Simulation of the contact problem was performed in the ANSYS 17.2, to refine the result taking into account the real boundary conditions. The problem is axisymmetric and half the cross section of the specimen considered. The load applied to the ball was set in accordance with the experimental data. Two types of boundary conditions were considered, in order to determine the effect of the hole for the microscope. In the first case, the fixing in the model were made along the contour of the hole in the steel protection, in accordance with the real experiment. In the second case, the bottom base of the model was fixed completely, in accordance with the conditions of the Hertz problem for a half-space [3]. Comparison of Huber's analytical solution with the numerical method for estimating the maximum tensile stresses on the surface in the vicinity of the contact area, taking into account the hole for the microscope, shows that the relative difference between solutions does not exceed 0.14%. When comparing the maximum tensile stresses obtained by the numerical method for the Hertz problem and for real specimens placed on the hole for the microscope in the experiment, the relative difference does not exceed 1.2%. Consequently, bending stresses from the hole do not significantly affect the tensile stresses in the vicinity of the contact area.

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DYNAMIC GROWTH OF A SELF-SIMILAR ANTI-PLANE CRACK DURING EARTHQUAKES

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We consider an anti-plane straight shear crack Γ lying along x-axis in the 2-D elastic domain of full space $\mathbb{R}^2$. The rigidity and the shear wave speed are assumed to be unity in the following. Let $\boldsymbol{u}(x,y,t)$ be the anti-plane displacement field defined at position $(x,y) \in \mathbb{R}^2 \backslash \Gamma$ and time $t \in \mathbb{R}_+$. In the anti-plane problem, the gap of displacement defined as

$$D(x,t) := \lim_{y \downarrow 0} u(x,y,t) - \lim_{y \uparrow 0} u(x,y,t), \quad x \in \Gamma$$

indicates slip of the elastic material along Γ. We also consider slip rate $V := \partial_t D$.

Seismology has revealed that earthquake faulting process shows self-similarity

$$\int_\Gamma D(x,t)\, dx \propto t^n \tag{1}$$

during its growing phase [e.g., Meier *et al.* 2016], where n is the dimension of the model space (i.e., $n = 2$ for our case and $n = 3$ for natural earthquakes). Simultaneously, Γ grows with 40-90% of the shear wave speed. Therefore, we assume that $\Gamma = (-vt, +vt)$, where $v \in (0,1)$ is the rupture velocity, and D and V are homogeneous functions of degree one and zero, respectively, to satisfy eq.(1);

$$D(x,t) = tD(X), \quad V(x,t) = V(X), \quad X := x/t. \tag{2}$$

With the assumption (2), we found that the shear stress perturbation, $T(x,t)$, due to the slip has the relationship with V as follows:

$$\frac{X}{\sqrt{1-X^2}} \frac{dT(X)}{dX} = \frac{d}{dX} \int_{-v}^{+v} \frac{V(\xi)}{X-\xi} \frac{d\xi}{\pi}, \quad x \in \Gamma. \tag{3}$$

Therefore, after integrating both side of eq.(3) with respect to X, we get the Airfoil equation [Tricomi 1957] for given $T(X)$.

T must balance with friction on the sliding crack surface. Given a result of a rock-friction experiment, we assume that friction is a decreasing function of slip and obtain the closed-form solution $V(X)$ that allows us to discuss physics and energetics of the crack problem with some physical requirements.

This work was funded by the Japan Society for the Promotion of Science (KAKENHI No. 17H02857).

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ON SOME PROBLEMS FOR FRACTURE PHENOMENA

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We discuss objectives of the Russia-Japan project to achieve in two years and plans to facilitate international collaboration, and propose the following subject matters mainly;

1. "**Dynamic contact on crack faces with friction**" Mathematical analysis for contact problems with friction has been well studied in the static case, e.g. [3, 5] etc. However, we still have some open problems in the dynamic case (cf. [1, 2, 8]). Then, we overview the related known results and clarify the main difficulty, moreover, we discuss the strategy for solving the problem and also the possibility of application to fault rupture in earthquakes.

2. "**Numerical simulation for crack problems in a nonlinear elastic model**" In [4] it is showed the existence of the generalized solution for a crack problem in a nonlinear elasticity by constructing an approximation problem. The feature of this model is that the stress concentration is allowed at the crack tip in the context of the infinitesimal strain. The main interest is to know the asymptotic behavior of the stress field near the crack tip in this model.

3. "**Bending elastic plate with a thin rigid inclusion**" In [6, 7], a problem of bending an elastic plate with a thin rigid inclusion both with the presence or absence of delamination is considered. Then, we try to derive the asymptotic behavior of solutions near the tip of the inclusion by using the complex stress function method.

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REMARKS ON THE REGULARITY OF NON-STATIONARY SLIP BOUNDARY VALUE PROBLEMS OF FRICTION TYPE

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We consider the non-stationary incompressible Navier–Stokes equations in a bounded smooth domain $\Omega \subset \mathbb{R}^d$ ($d = 2, 3$) as follows:

$$\begin{aligned} \partial_t u + u \cdot \nabla u - \nu \Delta u + \nabla p = f \qquad & \text{in} \quad \Omega \times (0, T), \\ \operatorname{div} u = 0 \qquad & \text{in} \quad \Omega \times (0, T), \\ u|_{t=0} = u_0 \qquad & \text{in} \quad \Omega \times (0, T), \end{aligned}$$

where $\partial\Omega = \Gamma_0 \cup \Gamma_1$ with $\overline{\Gamma}_0 \cap \overline{\Gamma}_1 = \emptyset$. We impose the no-slip boundary condition $u = 0$ on Γ_0 and the *slip boundary condition of friction type* proposed by [1] on Γ_1 (also known as given friction condition or Tresca condition):

$$u \cdot \nu = 0, \qquad |(\sigma\nu)_\tau| \le g, \qquad (\sigma\nu)_\tau \cdot u_\tau + g|u_\tau| = 0 \qquad \text{on} \quad \Gamma_1,$$

where $\sigma = -p\mathbb{I} + \nu(\nabla u + \nabla u^\top)$ denotes the stress tensor, ν is the outer unit normal to Γ_1, the subscripts τ mean the tangential parts of vectors, and $g > 0$ is a threshold of tangential stress. In [2] we proved (local-in-time) strong solvability of the above problem in the class

$$u \in C([0, T]; H^1(\Omega)^d), \quad \partial_t u \in L^2((0, T); H^1(\Omega)^d), \quad p \in L^\infty((0, T); L^2(\Omega)),$$

under the assumptions $u_0 \in H^2(\Omega)^d$ (plus compatibility conditions), $f \in H^1(0, T; L^2(\Omega)^d)$, and $g \in H^1((0, T); H^1(\Gamma))$. However, we did not know if this solution admits $u \in L^\infty((0, T); H^2(\Omega)^d)$, which holds, e.g., in case of the linear slip boundary condition.

In this talk, we prove that the higher regularity mentioned above actually holds true. The key idea for the proof is the H^2-estimate for the stationary equations $u \cdot \nabla u - \nu \Delta u + \nabla p = f$ based on methods of regularization (of friction terms), localization, and difference quotient. Moreover, if $d = 2$, we show that the strong solution in the maximal L^2 regularity class can also be constructed. The key for this observation is to combine the H^2-estimate above with a semi-discrete (w.r.t. time) problem.

If time permits, we also discuss the stationary Stokes problem with less regular friction parameter g than $H^1(\Gamma)$. For example, if $g \in H^{1/2}(\Gamma)$, then combining the methods of [3] and [4] yields $u \in H^{7/4-\epsilon}(\Omega)^d$. Finally, we would like to mention strong solutions to dynamic friction problems considered for linear isotropic elasticity (see [5, Section III.5.6]).

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MECHANICAL PROPERTIES OF A COMPOSITE MATERIAL OF CHITOSAN-VANCOMYCIN-NANOCELLULOSE NANOPARTICLES OF BACTERIAL ORIGIN TO CLOSE DURA MATER DEFECTS

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After the cranial injures the meninges of the human brain may require restoration, however, it is not always possible. For such cases it makes sense to synthesise a biocompatible material fit for a clinical use. Bacterial cellulose is widely used as a material for the treatment of burns and cosmetology. It is a nanoscale biopolymer that is close in its properties to collagen, obtained by culturing in a medium containing carbohydrate the bacteria Komagataeibacter xylinus under static conditions. Our product is distingushed by the presence of chemically and physically processed Chitosan with the antibacterial drug Vancomycin. The use of this material for the dura mater implant implies a strict

Ruptured sample after the experiment.

requirements to its mechanical properties. For this goal a number of mechanical experiments were performed (see the figure above) with samples of the material with different synthesis parameters.

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INVERSE PROBLEMS FOR ELASTIC BODIES WITH THIN INCLUSIONS

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Inverse problems for elastic bodies with thin elastic inclusions are analyzed. We assume that the inclusions cross the external boundary of the elastic bodies. A connection between the inclusions and the elastic bodies are characterized by a damage parameter. We investigate a dependence of solutions on the damage parameter. In particular, passages to infinity and to zero of the damage parameter are analyzed. Assuming that the damage and rigidity parameters of the model are unknown, inverse problems are formulated. Sufficient conditions for the inverse problems to have solutions are found. Estimates concerning solutions of the inverse problem are obtained. Local stability of solutions is established.

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NUMERICAL MODELLING OF THE OPTIMIZATION OF CEREBRAL BYPASSES

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A vascular bypass is a combination of usually separate vascular compartments. This technique is widely used in the treatment of cerebral aneurysms, when a vital parts of an artery have to be sacrificed with an anomaly during the operation. The purpose of the bypass formation is to preserve the blood flow rate of an occluded vessel. However, in some cases, the installation of a bypass can cause a number of complications after surgery. Thus, the question about the necessity of the formation of a bypass in a particular clinical case remains open.

In this paper, we consider 2 fundamentally different clinical outcomes in the formation of topologically closed bypasses. In the first case after the formation of the anastomosis there were no complications. As a result of the second operation, a thrombus formed in the stagnant zone appeared near the site of the anastomosis. This required an urgent reoperation. Using the ITK-Snap program, vascular geometry was restored before and after surgery in each considered clinical case [1]. In addition, we analyzed the approach described in [2] and found that the hydraulic criterion does not cover all possible cases of bypass formation and, therefore, is not applicable for the preoperative assessment of bypass quality.

Numerical FSI calculations and calculations in rigid-wall configurations were implemented in the ANSYS / CFX package in each of the reconstructed models, including alternative treatment models without formed bypass. We imposed boundary condition according to the data from the literature [3]. As a result of the study, it was possible to reproduce the real postoperative picture of the blood flow and the changes in the geometry of the vessels after the operation. The hemodynamic parameters of blood flow were also compared for the two clinical cases under consideration.

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ON THE CONNECTION BETWEEN TWO EQUILIBRIUM PROBLEMS FOR CRACKED TWO-DIMENSIONAL BODIES IN THE CASES OF THIN AND VOLUME RIGID INCLUSIONS

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We consider equilibrium problems for an inhomogeneous two-dimensional body with a crack and a rigid inclusion. The matrix of the body is assumed to be elastic. The boundary condition on the crack curve is an inequality describing mutual nonpenetration of the crack faces. We study two different equilibrium models. For the first model, we assume that a volume rigid inclusion is described by a domain. The second one describes a body containing a set of connected thin rigid inclusions, each corresponding to a curve. The crack is given by the same curve in both models. We prove that the solutions of equilibrium problems corresponding to the second model strongly converge to the problem solution for the first model as the number of inclusions tends to infinity, see [1].

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SMOOTHING AND STRICHARTZ ESTIMATES TO PERTURBED KLEIN-GORDON EQUATIONS IN EXTERIOR DOMAIN

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In this talk, we deal with the smoothing and Strichartz estimates to magnetic Klein-Gordon equations in exterior domain with time-dependent perturbations. Also, the smoothing estimates are applied to establish a scattering of solutions for small perturbations.

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A VARIATIONAL PROBLEM FOR AN ELASTIC BODY WITH SMALL PERIODICALLY LOCATED CRACKS

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In this work we consider the theory elasticity problem on a domain with periodically located cracks. It is supposed, that opposite faces of cracks can not penetrate into each other. The period of distribution of cracks, as well as their sizes, depends on the small parameter. The problem is non-linear, it results in a variational inequality in some convex closed set of admissible displacements.

The behavior of the solution of the problem with periodically located cracks is determined by the first two terms u^0, u^1 of the asymptotic expansion. Then $u^0(x)$ is the solution of the elasticity problem in a domain without cracks, and the first corrector $u^1(x, y)$ is the solution of the variational inequality for a given function $u^0(x)$ on a periodicity cell. The convergence of the problem solution to the solution u^0 of the homogeneous problem was proved in [1]. We proved complete convergence, using the periodic unfolding tools, introduced by [2].

For the first corrector $u^1(x, y)$, we construct an approximate equation, using the penalty and iteration methods. We prove that the sequence of the solutions of the problem with a penalty converges to the solution of the problem on the cell, when the small regularization parameter tends to zero. We show that the approximate solution of the iteration equation converges strongly to the solution of the penalty equation.

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HIERARCHY OF HYPERELASTIC MODELS IN ANEURYSM WALL MODELLING

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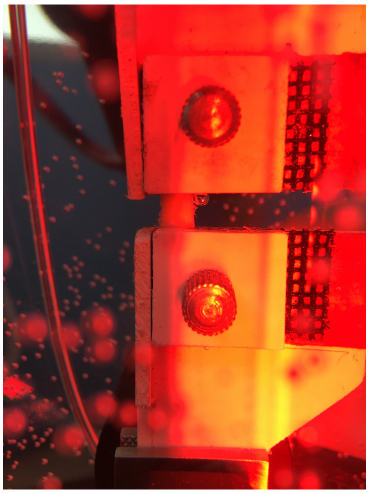
In the last decade preoperative modelling of intracranial aneurysm surgical treatment has undergo a drastic development, as it allows to predict the resulting outcome. Since risk of aneurysm rupture is rather small and comparable to the risk of development of postoperative complications it is a promising method that allows to individually assist every patient case. The key part of preoperative modelling is solving an FSI problem, where it becomes necessary to choose hyperelastic model for aneurysm wall. At the moment in the modern medicine the choice of model at that step is not dictated by any specific parameters of the considered case. The main task of this work was to compare different hyperelastic models and compare their applicability for aneurysm wall tissue modelling.



Specimen during the experiment.

For this study four hyperelastic models were chosen – 3 and 5 parameter Mooney-Rivlin models, Yeoh model and Neo-Hookean model. For base of the modelling we used the experimental data obtained during mechanical uniaxial test of the aneurysm tissue samples (see the figure above). Basing on the results of the mathematical modelling the hierarchy of the hyperelastic models was built, that shows what model would be optimal to use depending on the degree of deformation and aneurysm status.

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THE EFFECT OF SINGLE-WALLED NANOTUBES ON THE MECHANICAL PROPERTIES OF SILICONE COMPOSITES

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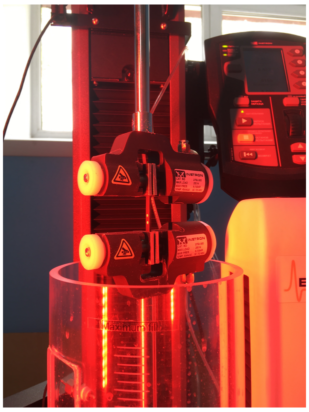
Carbon nanotube is the structure which can be widely used in creation of different materials as it provides unique properties. In particular, they are widely used in new electrically conductive devices [1,2,3]. The study of composite materials [4] is global in nature, caused by the explosion of the popularity of its usage in industry.

In this study we investigated the silicone composites with different content of nanotubes and at various stages of the composite moulding (from several days to several weeks). For mechanical testing universal tensile machine Instron 5944 was used (see the figure below). During the experiment, data on the relative elongation were recorded from 2 channels: according to the data of the traverse movement and using a video extensometer. The first results let us to talk about detecting differences in the strength properties of composites at different stages of moulding.



Material during the loading.

This study was funded by the Russian Foundation for Basic Research № 17-08-01736 and Russian government № 14.W03.31.0002

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JUNCTION PROBLEM FOR TIMOSHENKO AND SEMIRIGID INCLUSIONS EMBEDDED IN ELASTIC BODY

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In the paper, we consider a junction problem for a Timoshenko elastic inclusion and a semirigid thin inclusion located inside an elastic body, occupied the domain Ω. The terminology "semirigid" means that the inclusion is elastic in a given direction (and satisfies an equilibrium equation) being rigid in another direction, i.e.

$$-w_{,11} = [\sigma_\tau] \text{ on } \gamma;\ -\varphi_{,11} + v_{,1} + \varphi = 0, -v_{,11} - \varphi_{,1} = [\sigma_\nu] \text{ on } \gamma_t,$$

$$v|_{\gamma_s} \in L(\gamma_s),$$

where $[h]$ is a jump of a function h on γ; γ_t and γ_s are the Timoshenko and the semirigid parts of inclusion $\gamma = \gamma_t \cup \gamma_s$ respectively; v and w are vertical and horizontal displacements of inclusions; φ is the rotation angle within the Timoshenko beam model; σ_τ, σ_ν are the tangential and normal components of stress tensor; $L(\gamma_s) = \{l \mid l(x_1) = c_1 x_1 + c_0 \text{ on } \gamma_s;\ c_0, c_1 \in \mathbb{R}\}$.

The inclusions are assumed to be delaminated from elastic body, thus forming a crack between the inclusions and the elastic matrix. Nonlinear boundary conditions are imposed at the crack faces to prevent a mutual penetration between the crack faces:

$$[u_\nu] \geq 0 \text{ on } \gamma,$$

Here, u_ν is the normal displacements of the points of the elastic body. The inclusions have a joint point. In this point the suitable junction conditions can be writing in the following form

$$v_{,1}(0+) = -\varphi(0-);\ [v(0)] = 0, \int\limits_{\gamma_s} [\sigma_\nu] l + \varphi_{,1}(0-) l_{,1}(0+) - (v_{,1} + \varphi)(0-) l(0+) = 0\ \forall\, l \in L(\gamma_s).$$

A passage to a limit is investigated as a rigidity parameter of the elastic inclusion goes to infinity. The limit model is investigated.

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A CONTACT OF TWO ELASTIC PLATES CONTAINING THIN RIGID INCLUSIONS

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An equilibrium problem of two elastic plates located in parallel without a gap is considered. The plates of the same shape and size are clamped at their edges. To describe the displacements of all points of elastic plates the Kirchhoff-Love model is used. The deflections of the plates satisfy the nonpenetration condition. Each of the plates containes a thin rigid inclusion of the same location and shape. Those inclusions are connected to each other on all their length. It is assumed that an elastic force [1, 2] acts at the contact surfaces of the plates both in contact plane and orthogonal to it, and its value is characterized by a so-called damage parameter [1]. Two limit cases are studied when the damage parameter equals to zero or tends to infinity. The first case correspondes to contact without friction of two plates. The second one correspondes to equilibrium of two-layer plate containing a thin rigid inclusion.

The work was funded by the Russian Foundation for Basic Research according to the project № 19-015-50004.

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MODELING DESTRUCTION OF MONOLITHIC AND SPACED BARRIERS FROM ANISOTROPIC MATERIALS

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The last time the creation of materials with desired properties has become relevant. Modern technologies make it possible to optimize the structural strength parameters for working in certain modes of external impact. Such optimization can be done either by ordering of the material structure or by material reinforcement. In a composite material, the matrix and inclusions are assembled into a macroscopically multiphase material to enhance specific physical properties which are unreachable for the components individually. After this arrangement, the material acquires a high degree of anisotropy. Unfortunately, despite a large number of researches creating of materials with desired properties, there are only few developments relating to anisotropic materials. There is almost no data on the behavior of such materials under dynamic loads. This applies to both experimental research and mathematical modeling.

In addition to optimizing material properties, various approaches related to structural solutions are also used. For example, spaced obstacles are widely used to protect structures from impact.

In this research, we study the process of the development of destruction in monolithic and spaced barriers during high-speed interaction. The barrier material is orthotropic: organoplastics with a high degree of anisotropy of elastic and strength properties. The destruction and effectiveness of monolithic and spaced barriers were studied depending on the orientation of the properties of the anisotropic material in the range of impact velocities from 750 to 3000 m / s.

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ASYMPTOTIC MODELLING OF SOFT AND STIFF INTERFACES IN KIRCHHOFF-LOVE PLATE THEORY

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The characterization of the interface conditions between bonded together elastic media is a classical problem in solids mechanics. This problem arises in many fields of engineering when composite material should be modelled. For example, laminate structures can be built by gluing together thin plates. Due to the small thickness of glue layer (or adhesive) the numerical computation of the solution of the corresponding boundary value problem can be very difficult because it requires a fine mesh and, consequently, increasing of degree of freedom of the corresponding system.

In this situation, instead of the full model the approximate model with the interface condition between adherents is introduced. Many interface conditions are currently studied rather well from both mathematical and mechanical point of view for different models of solids mechanics: linear and nonlinear elasticity, piezoelectricity, magneto-electro-thermo-elasticity [1-5].

In the present work, we consider a composite structure consisting of two plates glued together by a third one (adhesive layer) along some common interfaces. The structure is in equilibrium under the action of applied forces. The composite plate is fixed on the parts of the external boundary. The equilibrium problem (case of pure bending) is formulated as a variational one. Namely, a minimization of the energy functional over a set of admissible deflections of the composite plate in the space H^2. It means that the deflections of each plates are described by biharmonic equations. And on the common boundaries the condition of equality of the deflections and their normal derivatives is satisfied. It is assumed that the elastic properties of the adhesive layer depend on its thickness ε as ε^N, $N \in \mathbb{R}$. Parameter ε is a small parameter of the problem. But the elastic properties of the glued plates do not depend on ε and remain constant.

The main goal of the work is to strictly mathematically justify the passage to the limit when ε tends to zero. It is shown that there are 7 limit problems in dependence on N. Moreover, for these 7 problems it is shown that the influence of adhesive on adherents can be replaced by interface.

Financial support is indicated at the end of the text. The work was funded by the Russian Foundation for Basic Research according to the project № 18-29-10007.

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A COHESIVE CRACK MODEL FOR PLATES

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Lavrentyev Institute of Hydrodynamics of SB RAS, Novosibirsk, Russia

We introduce a fully coupled variational model for elastic plates with interfacial cracks that accounts for the cohesive forces acting between the crack faces. The density of the energy spent by the cohesive forces depends on the crack opening and is not convex. We focus on the study of the differentiability of the reduced energy with respect to the crack length.

UNIQUENESS AND STABILITY FOR INVERSE HYPERBOLIC PROBLEMS BY AN INTERNAL OBSERVATION

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Let $\Omega \subset \mathbb{R}^n$ and $\omega \Subset \Omega$ be two domains. We consider the inverse source problem to determine F by some information of a solution on an internal domain ω for a hyperbolic equation $(\partial_t^2 - \mathcal{L})u = F(x,t)$, where $\mathcal{L}$ is a uniformly elliptic operator on Ω.

In this talk, we gain some results on uniqueness and stability under some assumptions on forms of the source term F, $\mathcal{L}$ and domains.

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INVERSE SCATTERING PROBLEMS ON METRIC GRAPHS

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I investigate inverse scattering problems for a Sturm-Liouvile operator on the metric graphs. The scattering matrix, part of the negative eigenvalues and corresponding normalizing coefficients are taken as a scattering data. The main goal of this research is to reconstruct the coefficients of Sturm-Liouvile operator on the basis of the given scattering data. I have deduced Marchenko equation which allowed us to prove the uniqueness theorems, provided a reconstruction procedure for the coefficients on the half-lines and investigated the conditional stability of the inverse scattering problem. Later on I have used the asymptotic behaviour of the scattering matrix to investigate the geometry of the graph. Most of the results of this presentation were obtained jointly with Prof. K.Mochizuki.

FLOW BEHAVIOR IN ELASTIC PATIENT-SPECIFIC CEREBRAL ANEURYSM

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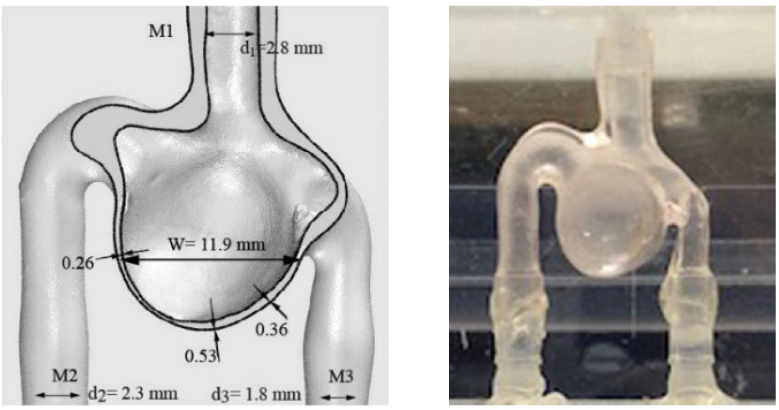
Introduction. The middle cerebral artery (MCA) is one of predilection vessels of aneurysm. In hemodynamics of cerebral aneurysm, a lot of researchers assumed to be rigid wall in CFD. A few numerical approaches were carried out for the cerebral aneurysm with elastic wall in FSI [1]. Particularly, there are few experimental studies with respect to hemodynamics for elastic cerebral aneurysm [2]. In the present study, the effect of wall elasticity on flow behavior was experimentally examined for the comparison of elastic with little deformable models in pulsatile flow using PIV.



Experiment and Results. The phantom morphology of full-scale patientspecific aneurysm initiated at the apex of the bifurcation at MCA is shown on the left figure.

Two outlet vessels $M2$ and $M3$ bifurcate from inlet vessel $M1$. The size of phantom model is the width of $W = 11.9$ mm and the depth of $D = 14.6$ mm, respectively. The thickness of dome is $\delta t = 0.38$ mm in average. The current phantom model with elastic modulus of $E = 0.67$ MPa was fabricated in silicone elastomer of Sylgard 184. The working fluid is aqueous glycerol sodium iodide solution with the same refractive index $n = 1.412$ as silicone. The properties of working fluid are density $\rho = 1238$ kg/m^3 and kinematic viscosity $\nu = 4.190 * 10 - 6$ m^2/s. The phantom model is horizontally set up acrylic bath shown on the right figure. In the pulsatile flow waveform, one period is 1.0 s, and the Womersley number α of 1.78, Re_{max}= 530 and $Re_{min} = 250$.

At peak systole, the flow behavior in elastic is moderate than that in little deformable models. After colliding the stagnation point at dome bottom the main flow circulates clockwise and separates. The particle trace in elastic is a little different from that of little deformable models. The particle trace in elastic model denotes several vortices, much complicated and phantom expands. The deformation in elastic is larger than that of little deformable models. With respect to width W, deformation ratios are 9% and 1.9 % in elastic and little deformable models, respectively. The deformation of elasticity might affect the flow behavior, i.e. velocity, wall shear stress and flow instability.

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