Neoclassical approach to objectivization of competency assessment

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Abstract. Hybridization of models for various methodological contexts are considered. This enables to improve the objectivization of competency assessment. The results obtained can be applied to building both educational competencies and those required by employers. The peculiarity of the methodology is supposed to be a risk-tolerant method used for improving the objectivity of competency assessment in the unit: teacher-student (expert-testee).

Keywords: hybridization, objectivization, mathematical modeling, competency assessment, entropy approach, fuzzy logic.

JEL Classification: C83.


Introduction

A more powerful tool for improving the objectivization of judgments and conclusions especially those of scientific nature is considered to be a mathematical method, mathematical modeling. The most important requirement for it is the one for adequacy of a character system chosen, namely, of a mathematical model to the object under study or some theoretical structure to be used for a theory explanation. This requirement is not often checked and the fact of using a mathematical method is assumed to be a result by default. It is fairly and accurately noticed that “a mountain of information collected and a scrupulously conducted modeling often gives birth to a mouse of a final cognitive result”. [1] A detailed answer to this matter of deep methodological properties needs more space to be published. In this paper the basic facts being responsible for the adequacy of those model structures involved for objectivization of professional competency assessment have been highlighted.

The adequacy of a mathematical model (MM) appears to depend on: critically sufficient qualitative description (study) of the object by the features chosen; reasonable quantitative description of the object by the features chosen; constant awareness of rational responsibility rather than deductive logic for the adequacy of MM.

If these aspirations of a researcher are achieved, it is appropriate to consider the quality of MM. It is the quality of MM that is the adequacy of the model together with its efficiency. In this case the concept of model adequacy is directly based on the definition of the modeling object and the notion of efficiency – on the modeling purposes [2].

The most common situation in the research experience is using a questionably adequate tool environment (a body of MM, computer support, information environment to be attainable in terms of data availability and interaction technology of these components) unless its limited explanatory and visual properties come to the surface. Then, an abrupt transition to a different environment or gentle combination with a new tooling environment takes place. And then one fairly relies on higher adequacy in eclectic. The situation is particularly prevalent among experienced researchers. Being at the level of their rich professional intuition they are capable of generating such eclecticism with a given set of consumer properties, in fact, working with hybrid models. The idea of hybridization of MMs of different classes is not new. It seemed to appear after the period of “naïve” mathematical modeling when proper use of mathematical tools in the essentially verbal sphere (retrospective experience of the humanities) was emphasized. But the awareness of semantics strengthening of the rational category of MM adequacy did not come immediately on the methodological field. It needed to gain sufficient MM experience and its awareness.

Here we can share our experience of constructing hybrid models from quite different subject areas relying on some methodological and methodical conclusions to a greater extent.

Hybridization of multivariate regression equations in a single MM adequately describing mechanical properties of factory rolled steel and its chemical composition is used as a constraint and target function in a linear programming problem at the next stage. Model effectiveness by this mixing has shown the 30% increase in the strength properties of steel maintaining the chemical composition on the model recommended levels within the State Standard (GOST) [3].

Hybridization in a single MM of the classical network model describing a large-scale investment process with an orthogonal regression model is like a convolution of specially organized, planned simulation experiments. The indicator of integral imbalance between the given investment dynamics and the one received during the experiment (the estimated dynamics) has been used as a response function. To vary the planning matrix of extreme experiments three groups of factors appearing to be sufficiently significant have been chosen: risk-factors...
In the formula (2) T – dichotomous thresholds of assessing indicators by components: $T_K$, $T_S$ and $T_D$ (or by each indicator: $T_{Ki}$, $i = 1,n_K$ , $T_{Sj}$, $j = 1,n_S$ and $T_{Di}$, $k = 1,n_P$) and also the threshold of competency acquisition $T_C$: the limit value of threshold values corresponds to the lower limit of «E» on the ECTS grading scale, i.e. to 5 scores on a 10-point scale. Under a more differentiated approach to the assessment, scaled acquisition thresholds: $T_{Ei}$, $T_{Di}$, $T_C$, $T_D$ and $T_E$, (threshold indexes are fixed on the ECTS grading scale) can be introduced. The grade $D_I$ shows the effectiveness of certain KSP indicators acquisition.

For every l-th competency the final grade of development $O_l$ and effectiveness $D_l$ have been calculated by the formulas (3) and (4):

$$O_l = \frac{10}{n} \sum_{i=1}^{n} D_{il} ,$$

(1)

$$D_l = \begin{cases} 0, & \text{if } O_l < T \\ 1, & \text{if } O_l \geq T \end{cases} .$$

(2)

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(3)

$$D_l = \frac{1}{n} \sum_{i=1}^{n} D_{il} ,$$

(4)

where $O_{il}$ and $D_{il}$ – a grade and dichotomous assessment of the i-th indicator for this competency component, calculated by the formulas (1) and (2); $n$ – a number of indicators of the l-th competency ($n \in N$).

However, it is obvious and well-known that components of a competency or those of an academic subject are interconnected and form a bonded structure. In this regard, it has been hypothesized that the quality of student performance within degree programs and competency development can be assessed with greater accuracy when assessing the bonds between the system elements rather than the elements themselves. In [7] the authors have suggested the following procedure of bond assessment between KSPs inside the competency:

Build a structural model of the competency consisting of KSPs and bonds between them.
Herewith, horizontal bonds between KSPs of the same category (Knowledge, Skill-Skill, Proficiency-Proficiency) can have mutually opposite bonds, and vertical – only direct bonds: Knowledge – Skill and Skill-Proficiency.

Assess KSPs on a 10-point scale by the formula (1) using a package of submission assignments.

Dichotomically assess «0» and «1» KSP acquisition by the formula (2) given a level of development as a set of threshold values $T_i$ for KSPs.

Calculate the effectiveness of KSP acquisition included in the $i$-th competency by the formula (4).

Based on the dichotomous assessment of KSP, assess the bonds with the pairs: «00», «01», «10», «11» where the left digit corresponds to the dichotomous assessment of KSP outflow element, and the right one – to the dichotomous assessment of KSP inflow element.

**Findings**

Due to the application of this algorithm to structuring the professional competency PC-20 «The ability to implement and support the choice of design solutions by types of information systems options» in the training program 09.03.03 "Applied Information Science" the structure presented in Fig. 1 has been obtained.

![Fig.1 – Structural model of the competency as a directed graph (12 KSPs, 63 bonds, 40 vertical ones and 4 bonds out of 23 horizontal bonds shown)](image)

In this configuration the competency represents a system of elements connected between themselves thus forming a unit (a single whole). Following this approach, a correct assessment of internal structure ordering can be made. On the basis of frequency distributions of dichotomous pairs, Shannon entropy determining competency development has been received. So, for the $i$-th competency the entropy of its internal structure for KSPs $H_i$ has been defined as follows:

$$H_i = -P_{00i} \log_2(P_{00i}) - P_{01i} \log_2(P_{01i}) - P_{10i} \log_2(P_{10i}) - P_{11i} \log_2(P_{11i})$$

(5)

where $P_{00i}$, $P_{01i}$, $P_{10i}$, $P_{11i}$ – relative frequency for occurrence of the pairs «00», «01», «10» and «11» within the dichotomous set of bonds assessment between KSPs. The entropy calculated is a measure of uncertainty regarding the competency development as a system.

Maximum entropy calculated by Hartley formula gives $H_{MAX} = 2$ for the system of four pairs, and thus, the entropy for competencies irrespective of the bonds in their structure are incommensurable. System specification $R$ (6) being free from this deficiency is always located on the segment [0, 1], regardless the number of elements in the system, assessment scale and logarithm base. Ordering assessment $R_i$ for the structural model of the $i$-th competency is made by the formula:

$$R_i = 1 - \frac{H_i}{2},$$

(6)

where $H_i$ – entropy of the competency internal structure by the formula (5).

The development of each competency is defined in Table 1 on the basis of $D$ effectiveness and $R$ ordering values (Table 1, in bold). When interpreting these values V. Bespalko recommends to put grades on a 5-point scale – «good» at effectiveness coefficient $D$ – 0,8…0,9 and «excellent» – 0,9…1,0. Therefore, the entropy value – 0,7 can be considered to be the border systemic criterion of the competency. This value corresponds to the average level in Table 1, i.e. to 90% effectiveness dichotomically assessed for demonstrable knowledge, skill, proficiency, and also to the threshold value of ordering – 0,3. The application of this mechanism for systematization of competencies sets a universal scale (according to V. Bespalko) for assessing individual learning achievements presented in Table 1. Scanning of the table should be carried out from top to bottom when assessing. If all the conditions in a line are met, an appropriate level is set. Otherwise, the competency is not developed.

**Table 1 – Universal scale for assessing individual learning achievements**

<table>
<thead>
<tr>
<th>D</th>
<th>H</th>
<th>R</th>
<th>Level of development</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 0,91$</td>
<td>$\leq 0,44$</td>
<td>$\geq 0,56$</td>
<td>5</td>
</tr>
<tr>
<td>$\geq 0,90$</td>
<td>$\leq 0,70$</td>
<td>$\geq 0,30$</td>
<td>4</td>
</tr>
<tr>
<td>$\geq 0,71$</td>
<td>$\leq 0,87$</td>
<td>$\geq 0,13$</td>
<td>3</td>
</tr>
<tr>
<td>$&lt; 0,71$</td>
<td>$&gt; 0,87$</td>
<td>$&lt; 0,13$</td>
<td>2</td>
</tr>
</tbody>
</table>

Performed experimental computer implementation of this technique shows the
advantage of taking into account the relationships between the elements of the competencies. In particular, it has been found out that the establishment of relations between KSPs and considering them when assessing the quality of student performance within academic courses ensures structural and logical integrity and improves the ordering of course content as well as increases the accuracy of competency assessment.

The appropriateness of including students' personal qualities in the competency structure is further shown. The formal procedure of structural modeling of educational programs and its components with due account of personal qualities has been developed. This procedure is that the associated structure of competency components is constructed as a set of a bimodal directed graphs

\[ M = \{ g((\Theta_k, \Theta_i), \Sigma_{kl}) \in G \} \]

where \( \Theta_k \) and \( \Theta_i \) – a pair of connected KSPs (\( k \neq i \)), \( \Sigma_{kl} \) – oriented bond from \( \Theta_k \) to \( \Theta_i \), \( g \) – a bimodal graph with one edge, \( G \) – a set of elements \( g \) forming a structural model of a competency.

So, the structure of the i-th competency includes a set of basic KSPs \( \Theta \beta \) (providing external bonds to other competencies) being disjoint with a set of basic KSPs \( \Theta \alpha \) (having only internal bonds), a set of personal qualities \( \Lambda \) determining its development in which a subset of competencies \( \Lambda \beta \) with their dichotomous assessment being equal to «1», and also its corresponding subsets of bonds \( \Sigma_{kl} \alpha \alpha_{kl} \) and \( \Sigma_{kl} \alpha \beta_{kl} \) between the k-th personal quality of the subset \( \Lambda_k \) and the i-th forming KSPs of the subset \( \Theta_i \) can be singled out:

\[
M_i = \{ g_1((\Lambda_i \delta_i, \Theta_i \alpha_i), \Sigma_{kl} \alpha \alpha_{kl}), g_0(\Theta_i \alpha_i, \Sigma_{kl} \alpha \beta_{kl}) \}
\]

where \( M_i \) – a structural model of the i-th competency, \( \Theta_i \beta_i \) and \( \Theta_i \alpha_i \) – a pair of connected basic and forming KSPs, \( \Sigma_{kl} \alpha \alpha_{kl} \) – oriented bond from \( \Theta_i \alpha_i \) to \( \Theta_i \alpha_i \), \( \Sigma_{kl} \alpha \beta_{kl} \) – oriented bond from \( \Theta_i \beta_i \) to \( \Theta_i \alpha_i \), \( g_0 \) – a bimodal directed graph, connecting basic and forming KSPs, \( \Sigma_g \alpha \beta \) – inflow external bond between basic KSPs of some j-th competency and basic KSPs of the i-th competency, \( \Sigma_g \alpha \beta \) – outflow external bond between forming KSPs of the given i-th competency and basic KSPs of some j-th competency; \( \Lambda_i \delta_i \) and \( \Lambda_i \alpha_i \) – dominant and forming PQs having oriented bonds with forming KSPs \( \Theta_i \alpha_i \) – correspondingly, \( \Sigma \delta \alpha_{kl} \) and \( \Sigma \alpha \alpha_{kl} \) (opposite direction being impossible), \( g_1 \) and \( g_2 \) – bimodal directed graphs connecting dominant and forming PQs correspondingly and forming KSPs. The model (7) is graphically presented in Fig. 2.

Dichotomous values of PQs are similarly defined to dichotomous values of KSPs with thresholds. Therefore, available values for assessing bonds \( \Sigma_{kl} \alpha \alpha_{kl} \), taking into account only dominant PQs (bonds \( \Sigma_{kl} \delta \alpha_{kl} \) in Fig. 2), are «10» and «11». So, the formula for calculating the entropy \( H_{\Lambda_i} \) of the subsystem «KSPs–KSPs» of the i-th competency is as follows:

\[
H_{\Lambda_i} = -p_{\Lambda_{10i}} \log_2(p_{\Lambda_{10i}}) - p_{\Lambda_{11i}} \log_2(p_{\Lambda_{11i}})
\]

where \( p_{\Lambda_{10i}}, p_{\Lambda_{11i}} \) – probabilities for appearance of the values «10» and «11» within a set of bond values \( \Sigma \alpha \beta \) between the elements PQs and KSPs of the i-th competency.

![Figure 2 – System representation of educational competencies](image)

In this case the formula (6) for calculating the conditional probability of dichotomous values of PQ occurrence \( \Lambda_i \) of the i-th competency (5); \( p_{\Lambda_{11i}} \) – the conditional probability of dichotomous values of PQ occurrence \( \Lambda_i \) of the i-th competency, in the denominator – maximum entropy of the combined system of «KSP–KSP» and «PQ–KSP» in the limiting case of their independence.

The formula for calculating efficiency of the i-th competency (4), which is a probability of dichotomous value occurrence «1» among KSPs for the combined system of «KSP–KSP» and «PQ–KSP», takes an averaged value:

where \( D_{\theta_{\Lambda_i}} \) – efficiency on KSPs (4), \( D_{\Lambda_i} \) –
similarly calculated efficiency on PQs:

\[ D_i = \frac{(D_{qi} + D_{Di})}{2}, \tag{10} \]

The given approach provides a means of applying the systemacity principle to structuring academic courses and degree programs and also implementing a methodology developed for the objective quality assessment of student performance within degree programs.

Consequently, the entropy approach to competency assessment by taking into account its structure and calculating ordering which reduces the subjectivity of assessment has been further developed. The methods of assessing the quality of student performance within academic courses have also been proposed and its experimental computer realization confirming the advantage of taking into account the bonds between the competency elements has been performed.

Then, methods of fuzzy assessment of PQs on the basis of Leonhard-Smishek’s characterological questionnaire for a more complete objectivization of assessing competency development have been suggested, which uses expert assessment of conformity of the individual parameters according to Leonhard and PQs providing expert assessment of a certain set of PQs for the time duration of a typical test (1 hour). The usefulness of this technique is based on the fact that quantitative expression of a personality type in its limit value defines the so-called accentuation. K. Leonhard identified 10 personality accentuations (types), each integrates a set of PQs and is quantitatively assessed using a questionnaire-test developed in 1970 by H. Smishek. The nominal time duration of the test is 10 minutes. To match 39 PQs expertly selected and 10 personality types the method of expert assessment and fuzzy logic have been applied. It has been considered in detail in [8].

The studies of the methods for personality quality assessment have been carried out in the fourth year of Information Science and Computer Engineering Faculty, SibSUTIS (108 students) [8]. PQ assessment results for one testee are given in Table 2. The assessment obtained using the methods of fuzzy logic («FL assessment» column) is presented in the table, and also PQ assessment obtained using standard tests («Standard tests» column) is given. The results (Table 2) show that basic data correction has reduced deviation to 8%, which is 12% less than the assessment deviation without correction.

Table 2 – Personal quality assessment results with and without basic data correction, testee 1

<table>
<thead>
<tr>
<th>Personal quality</th>
<th>Without basic data correction</th>
<th>With basic data correction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FL assessment</td>
<td>Standard tests</td>
</tr>
<tr>
<td>Self-esteem</td>
<td>8,3</td>
<td>5,9</td>
</tr>
</tbody>
</table>

The final stage of this study represents development and functional study of a modeling-instrumental complex (MIC) implementing the methods for objectivization of assessing the quality of student performance within degree programs in the educational organization. The MIC information model as a database containing all necessary relevant data for solving MIC issues and expert knowledge has been developed. Being a window interface the software for MIC implementation is presented in two main forms: «Competencies guide» which gives tools for easy competency structuring and «Testing control module» which provides problem solving of testing and their results processing including calculation and output of final results and reports [9].

Conclusions

The results of these studies on assessment objectivization can draw some conclusions. Competency is considered to be a dynamic combination of knowledge, skill and proficiency, personal qualities to be taken into account when assessing. Searching for assessment and data analysis methods and models under uncertainty conditions has made it possible to determine the formal apparatus for improving the model adequacy for the research objectives.

The system approach to competency assessment taking into account its internal structure has been developed and studied thus reducing the degree of subjectivity in assessing. The technique of assessing the quality of student performance within academic courses has been proposed. It provides the advantage of taking competency element bonds into account. Considering these relationships (bonds) provides structural and logical integrity and improves the content ordering of academic courses, and it also increases the accuracy of assessing competency development.

Summing it up, it should be noted that in the humanitarian field the adequacy of modeling is achieved by a kind of combination of models and
methods of their various classes.

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**References**


