INVERSE PROBLEMS FOR A THREE-DIMENSIONAL EQUATION OF PARABOLIC-HYPERBOLIC TYPE IN FINDING TIME-DEPENDENT FACTORS OF THE RIGHT-HAND SIDES

Sidorov S.N.
Ufa State Oil Technical University, Ufa
stsid@mail.ru

Consider an equation of mixed parabolic-hyperbolic type $Lu = F(x, y, t)$, here

$$Lu = \begin{cases} u_t - u_{xx} - u_{yy} + bu, & t > 0, \\ u_{tt} - u_{xx} - u_{yy} + bu, & t < 0, \end{cases} \quad F(x, y, t) = \begin{cases} f_1(x, y)g_1(t), & t > 0, \\ f_2(x, y)g_2(t), & t < 0, \end{cases}$$

in the domain of $Q = \{(x, y, t) | \, (x, y) \in D, \, t \in (-\alpha, \beta)\}$, $D = \{(x, y) | \, 0 < x < p, \, 0 < y < q\}$, α, β, p, q are given positive real numbers, b is any given real number, and we set the following problems.

Problem 1. *Find functions $u(x, y, t)$ and $g_1(t)$ satisfying the conditions*

$$u(x, y, t) \in C(\overline{Q}) \cap C_t^1(Q) \cap C_{x,y}^1(\overline{Q}) \cap C_{x,y}^2(Q_+) \cap C^2(Q_-); \tag{1}$$

$$Lu(x, y, t) \equiv F(x, y, t), \quad (x, y, t) \in Q_+ \cup Q_-; \tag{2}$$

$$u(x, y, t)\big|_{x=0} = u(x, y, t)\big|_{x=p} = u(x, y, t)\big|_{y=0} = u(x, y, t)\big|_{y=q} = 0, \ -\alpha \le t \le \beta; \tag{3}$$

$$u(x, y, t)\big|_{t=-\alpha} = \psi(x, y), \quad (x, y) \in \overline{D}, \tag{4}$$

$$g_1(t) \in C[0, \beta]; \quad u(x_0, y_0, t) = h_1(t), \quad (x_0, y_0) \in D, \quad 0 \le t \le \beta, \tag{5}$$

where $f_i(x, y)$, $i = 1, 2$, $\psi(x, y)$, $g_2(t)$ and $h_1(t)$ are given functions, $Q_- = Q \cap \{t < 0\}$, $Q_+ = Q \cap \{t > 0\}$.

Problem 2. *Find functions $u(x, y, t)$ and $g_2(t)$ satisfying the conditions* (1) – (4) *and*

$$g_2(t) \in C[-\alpha, 0]; \quad u(x_0, y_0, t) = h_2(t), \quad (x_0, y_0) \in D, \quad -\alpha \le t \le 0, \tag{6}$$

where $f_i(x, y)$, $i = 1, 2$, $\psi(x, y)$, $g_1(t)$ and $h_2(t)$ are given functions.

Problem 3. *Find functions $u(x, y, t)$, $g_1(t)$ and $g_2(t)$ satisfying the conditions* (1) – (6).

The report will present uniqueness and existence theorems for solutions to Problems 1 – 3, while the solutions are constructed explicitly. These results are presented in [1], which has been accepted for publication.

REFERENCES

1. *Sidorov S.N.* Inverse problems for a three-dimensional equation of parabolic-hyperbolic type in finding time-dependent factors of the right-hand sides // Lobachevskii Journal of Mathematics (accepted for publication).