ACCOUNTING FOR ROUND-OFF ERRORS WHEN USING GRADIENT MINIMIZATION METHODS

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When solving applied inverse problems, it often becomes necessary to minimize some target functionals. Iterative methods are usually used for minimization. If the problem is linear, then one of the most common minimization methods employed is the conjugate gradient method. If the number of components in the required element realizing the minimum of the functional is N, then the conjugate gradient method converges to the exact solution of the problem in exactly N iterations. However, this statement is only true on the condition that all calculations are performed accurately and that there are no rounding errors. Nevertheless, when solving real applied problems, rounding errors can greatly affect the resulting approximate solution. Two cases are possible: in the first case, the value of the minimized functional becomes comparable to the background of rounding errors at some iteration, the number of which is less than N. Starting from this iteration, the value of the functional stops decreasing. This means that starting from this iteration, all subsequent calculations will not improve the solution and are meaningless. Therefore, a reasonable question arises — is it possible to track this moment in order to save computing resources? A positive answer to this question is useful, but not critical in solving real applied problems. On the other hand, the second case is essential for practice. In the second case, due to rounding errors in determining the minimization directions and the steps along them, it turns out that after performing N iterations, the value of the minimized functional is still quite large. This means that the found approximate solution can still be refined if the iterative process is continued. It turns out that the continuation of the iterative process will allow us to find a better approximation for the true solution. We emphasize that here, in contrast to the first case, the classical criterion for stopping the iterative process (by a fixed number of iterations equal to N) gives a bad result. As a result, there was a need to develop a method for automatically determining the number of iterations, in which the value of the minimized functional becomes comparable to the background of rounding errors [1]. Therefore, taking into account rounding errors when choosing a criterion to stop the iterative process is a relevant issue and in demand in practice.

REFERENCES

1. *Lukyanenko D., Shinkarev V., Yagola A.* Accounting for round-off errors when using gradient minimization methods // Algorithms, 2022, V. 15, no. 9, P. 324.