PROPERTIES OF THE NORMAL RADON TRANSFORM ACTING ON 3D SYMMETRIC m-TENSOR FIELDS

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The longitudinal ray transform is a classical integral geometry operator acting on vector and symmetric m-tensor fields [1]. In $\mathbb{R}^3$, only the solenoidal part of a vector or symmetric m-tensor field ($m \geqslant 2$) can be reconstructed from the longitudinal ray transform. To restore the potential part, we must have other data. One of operators that makes it possible to reconstruct the potential part of a tensor field is the normal Radon transform. In [2] the problem of inverting the Radon transform of a symmetric 2-tensor field and, in particular, the normal Radon transform was investigated. We note the papers in which the singular value decompositions of the Radon transform [3] and the normal Radon transforms [4, 5, 6] were obtained.

Let $B = \{x : |x| < 1\}$ denote an open unit ball in $\mathbb{R}^3$ and $Z = \{(s, \xi) : s \in (-1, 1),\ \xi \in \mathbb{R}^3,\ |\xi| = 1\}$ be a cylinder. The plane $P_{\xi,s}$ in $\mathbb{R}^3$ can be given by the normal equation $\langle \xi, x\rangle - s = 0$ for $x, \xi \in \mathbb{R}^3$, $|\xi| = 1$.

The normal Radon transform $\mathcal{R}_m^\perp : L_2(S^m(B)) \to L_2(Z, \rho)$ of a symmetric m-tensor field $\mathbf{u}(x)$ is defined by the formula

$$[\mathcal{R}_m^\perp \mathbf{u}](s, \xi) = \int\limits_{P_{\xi,s}} \sum_{i_1,\dots,i_m=1}^{3} \xi_{i_1} \dots \xi_{i_m} u_{i_1 \dots i_m}(x) dx.$$

In the report, we consider the integral geometry problem of finding a potential part of 3D symmetric m-tensor field provided that the normal Radon transform is known. Properties of the normal Radon transform acting on 3D tensor fields are investigated.

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