The mathematical relations associated with the vortex beam tomography

Balandin A.L.
V.M. Matrosov Institute of System Dynamics and Control Theory, Siberian Branch of the Russian Academy of Sciences, Lermontov str. 134, 664033, Irkutsk-33
balandin@icc.ru

The most imaging techniques use fields or waves to explore an object. The purpose of inverse scattering problems is attempts to infer about the object's parameters from the perturbed or the scattered fields induced by the presence of the object. The term vortex beam refers to a beam of electromagnetic radiation, electrons, photons, etc., whose phase changes in a corkscrew-like manner in the direction of propagation. The scattered wave carries internal structural information about the object by its amplitude and phase, wherein to resolution is an important indicator of the reconstruction quality. The perfect Bessel beams are the solutions of Helmholtz equation and can oscillate faster than their band-limited Fourier transforms suggest[1]. In cylindrical coordinates, the Bessel beams of integer order n are written as [2]

$$U_n^B(\rho,\varphi,z) = J_n(k_\perp\rho)\,\mathbf{e}^{in\varphi}\,\mathbf{e}^{ik_{||}z}, \tag{1}$$

where $J_n(k_\perp r)$ is nth-order Bessel function, φ is azimuth angle in the plane perpendicular to the direction $\hat{\boldsymbol{z}}$, $\boldsymbol{k} = \boldsymbol{k}_\perp + k_{||}\hat{\boldsymbol{z}}$, $k_\perp = |\boldsymbol{k}_\perp|$, $\boldsymbol{r}_\perp = (\rho\cos\varphi, \rho\sin\varphi)$.
The Bessel beam with fractional order, $U_\nu^B(\boldsymbol{r})$, can be expanded in terms of fundamental nth-order Bessel beams $U_n^B(\boldsymbol{r})$ as follows

$$U_\nu^B(\rho,\varphi,z) = \frac{\mathbf{e}^{i\pi\nu}\sin(\pi\nu)}{\pi}\sum_{n=-\infty}^{\infty}\frac{U_n^B(\boldsymbol{r})}{\nu-n}, \quad \nu \text{ is non-integer.} \tag{2}$$

The scattering potential $V(\boldsymbol{r})$ in the Born approximation is defined according to the equation

$$(\nabla^2 + k^2)U(\boldsymbol{r}) = V(\boldsymbol{r})U(\boldsymbol{r}), \quad V(\boldsymbol{r}) = -k^2\big(\varepsilon(\boldsymbol{r}) - 1\big), \quad k^2 = k_\perp^2 + k_{||}^2, \tag{3}$$

where $\varepsilon(\boldsymbol{r})$ is the electromagnetic permittivity of the media.
The relations connecting the scattering field and the scattering potential in Fourier space are obtained in the explicit form.

$$\tilde{U}{}^{n}_{sc}(\boldsymbol{q}_\perp, z_0) = \frac{i^{n+1}\,\mathbf{e}^{in\phi}}{2\pi\ k_\perp}\,\frac{\sqrt{\pi/2}\,\mathbf{e}^{ipz_0}}{p}\,\tilde{V}(\boldsymbol{q}_\perp - \boldsymbol{k}_\perp, k_{||} - p), \tag{4}$$

$$\tilde{U}{}^{\nu}_{sc}(\boldsymbol{q}_\perp, z_0) = \frac{\mathbf{e}^{ik_{||}z}\,\mathbf{e}^{i\pi\nu}\sin(\pi\nu)}{\pi}\,\frac{\sqrt{\pi/2}\,\mathbf{e}^{ipz_0}}{p}\,\frac{\tilde{V}(\boldsymbol{q}_\perp - \boldsymbol{k}_\perp, k_{||} - p)}{k_\perp}\sum_{n=-\infty}^{\infty}\frac{i^n\,\mathbf{e}^{in\phi}}{\nu-n}, \tag{5}$$

REFERENCES

1. *Berry M.V.* A note on superoscillations associated with Bessel beams // Journal of Optics, **15**, P. 044006(5pp),2013. doi:10.1088/2040-8978/15/4/044006.
2. *Gbur G.J.* Singular Optics, CRC Press Teylor & Fracis Group, New York, 2017.