

## Carleman Estimates for Mean Field Games

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In this talk, we will present our recent results of [7]-[13].

The mean field games (MFG) theory is a relatively new field, which studies the collective behavior of large populations of rational decision-makers. This theory was first introduced in 2006-2007 in seminal publications of Lasry and Lions [14] as well as of Huang, Caines and Malhamé [4]. Social sciences enjoy a rapidly increasing role in the modern society. Therefore, mathematical modeling of social phenomena can potentially provide a quite important societal impact. In this regard, the MFG theory is the single mathematical model of social processes, which is based on an universal system of coupled Partial Differential Equations (PDEs) [1, 3]. That system is the so-called Mean Field Games system (MFGS). The number of applications of the MFGS to the societal problems is flourishing and includes such areas as, e.g. finance, fight with corruption, cybersecurity, election dynamics, robotic control, etc.

Thus, due to a broad range of applications of the MFGS, it is important to address various mathematical questions for this system. In the series of seven recent publications in 2023 the presenter with co-authors has addressed questions of uniqueness and stability of various forward and inverse problems for the MFGS [7]-[13]. More precisely, Hölder and Lipschitz stability results are proven for these problems. They imply uniqueness. In addition, a globally convergent so-called "convexification" numerical method is developed for a forward problem for the MFGS. We use the term "forward problem" in the case when the coefficients of the MFGS are known and it is required to determine the solution of the MFGS using some initial, terminal and boundary conditions. We use the term "Coefficient Inverse Problem" if it is required to determine a coefficient of the MFGS, given Dirichlet and Neumann data at a part of the boundary and some initial and terminal conditions.

All results of [7]-[13] are obtained using the apparatus of Carleman estimates. Historically, Carleman estimates were first introduced in the field of Coefficient Inverse Problems in the work of Bukhgeim-Klibanov in 1981 [2]. The framework of [2] has been broadly used since then for proofs of global uniqueness and stability results for Coefficient Inverse Problems, see, e.g. [5, 6] and references cited therein for some follow up publications.

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