ON DEEP VOLATILITY CALIBRATION WITH NON-CLASSICAL VARIATIONAL PRINCIPLE

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The problem of volatility function calibration is an example of inverse coefficient problem [1] for Black-Scholes partial differential equation (PDE) [2, 3]. This inverse problem can be approached using various methods [4].

One of the ways to solve direct and inverse problems for PDE is the neural network approach, when the solution to the problem is approximated by an artificial neural network.

One of the common approaches to training such neural networks is minimizing the residual functional, however, if the boundary value problem admits a classical variational formulation, then as an alternative, the functional from the variational principle can be used for minimization [5], and if the classical variational principle does not exist, then one can try to construct a non-classical variational principle, following [6].

Black-Scholes equation is a parabolic type PDE and, for the case of constant volatility, it can be transformed into the heat PDE via change of dependent and independent variables. As is well known, no classical variational principle for the heat equation exists [7], but several varieties of non-classical variational principles can be constructed, including a convolution-based variational principle [8].

We demonstrate that a non-classical variational formulation with convolution can be constructed for Black-Scholes PDE with volatility function introduced in [9]. Neural network approximation for the solutions of Black-Scholes PDE for this local volatility model was reported in [10]. We discuss application of the functional from non-classical variational formulation as a loss functional for gradient descent and as a quality metric to prevent overfitting.

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