NUMERICAL STUDY OF THE EFFECT OF THERMOCAPILLARY FORCES AND ADDITIONAL TANGENTIAL STRESSES ON THE TEMPERATURE DISTRIBUTION IN THE FREE LIQUID LAYER

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Dynamics of flat layer of viscous non-compressible liquid with free borders and process of heat transfer in it under conditions of weightlessness are investigated. Free boundaries are subject to thermocapillary forces and additional tangential stresses. The dynamics of the liquid layer is modeled using exact solutions of the Navier-Stokes equations of a special form and a numerical algorithm of the "predictor-corrector" type. We consider the initial-boundary value problem in a three-dimensional formulation to calculate the temperature distribution. The stabilizing corrections method to calculate in a parallelepiped is considered. "Soft" conditions are formulated on the artificially introduced "vertical" ends of the computational domain. This conditions are a consequence of the heat transfer equation and conditions for temperature at infinity. The results of numerical experiments on the dynamics of a liquid layer and the process of heat transfer in it are presented in the case of different time-dependent functions that specify the temperature distribution and additional shear stresses at free boundaries.

Mathematical modeling of the dynamics of a free liquid layer and heat transfer in it

The Navier-Stokes and heat transfer equations

$$u_t + uu_x + vu_y + wu_z = -p_x + \frac{1}{\mathrm{Re}}\left(u_{xx} + u_{yy} + u_{zz}\right),$$

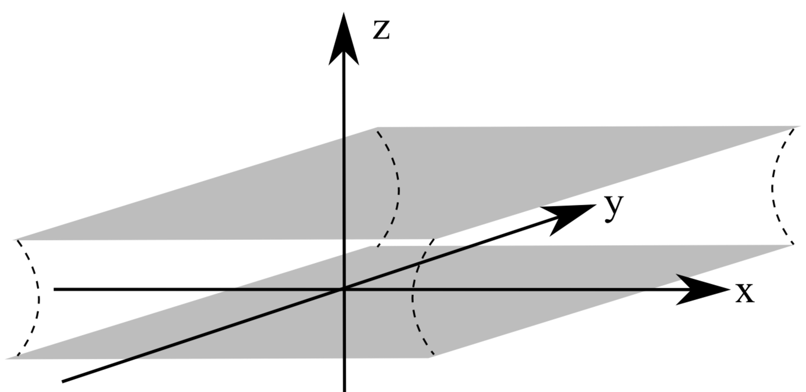
$$v_t + uv_x + vv_y + wv_z = -p_y + \frac{1}{\mathrm{Re}}\left(v_{xx} + v_{yy} + v_{zz}\right),$$

$$w_t + uw_x + vw_y + ww_z = -p_z + \frac{1}{\mathrm{Re}}\left(w_{xx} + w_{yy} + w_{zz}\right),$$

$$u_x + v_y + w_z = 0,$$

$$T_t + uT_x + vT_y + wT_z = \frac{1}{\mathrm{Re\,Pr}}\left(T_{xx} + T_{yy} + T_{zz}\right).$$

Geometry of the flow domain



Pukhnachev V.V., 1999

The solutions of the equations

Definition of the velocity components

$$u(x,z,t) = (f(t,z) + g(t,z))x,$$

$$v(y,z,t) = (f(t,z) - g(t,z))y,$$

$$w(x,z,t) = -2\int_0^z f(\alpha,t)\,d\alpha.$$

The problem of finding of a free boundary position

$$\frac{dZ}{dt} = -\int_0^{Z(t)} f(t,z)\,dz, \qquad Z(0) = Z_0 > 0.$$

The problem of finding functions *f* and *g* in a half of the layer

$$f_t + f^2 + g^2 - 2f_z\int_0^z f(\alpha,t)\,d\alpha - \frac{1}{Re}f_{zz} = 0, \quad g_t + 2fg - 2g_z\int_0^z f(\alpha,t)\,d\alpha - \frac{1}{Re}g_{zz} = 0,$$

$$f_z\left(Z(t),t\right) = \tilde{\tau}(t) - \frac{Ma}{2\,\mathrm{Re\,Pr}}\left(A(t) + B(t)\right), \quad g_z\left(Z(t),t\right) = -\frac{Ma}{2\,\mathrm{Re\,Pr}}\left(A(t) + B(t)\right),$$

$$f(z,0) = 0,\ f_z(0,t) = 0; \qquad g(z,0) = 0,\ g_z(0,t) = 0.$$

Boundary conditions

Dynamic conditions

$$-p + \frac{2}{\mathrm{Re}}\mathbf{n}\cdot\mathbf{D}(\mathbf{v})\mathbf{n}\,|_{z=\pm Z(t)} = -P_g,$$

$$2\mathbf{s}_1\cdot\mathbf{D}(\mathbf{v})\mathbf{n}\,|_{z=\pm Z(t)} = \tau_1(x,t) - \frac{Ma}{\mathrm{Re\,Pr}}T_x,$$

$$2\mathbf{s}_2\cdot\mathbf{D}(\mathbf{v})\mathbf{n}\,|_{z=\pm Z(t)} = \tau_2(x,t) - \frac{Ma}{\mathrm{Re\,Pr}}T_y,$$

$$P_g = \overline{P}_g - \frac{2}{Re}\overline{\rho}\,\overline{\nu}\,\mathbf{n}\cdot\mathbf{D}(\mathbf{v_g})\mathbf{n}\Big|_{z=\pm Z(t)}.$$

Kinematic condition

$$\mathbf{v}\cdot\mathbf{n} = \pm w|_{z=\pm Z(t)} = \frac{dZ}{dt},$$

Temperature distribution

$$T(x,y,\pm Z(t),t) = A(t)\frac{x^2}{2} + B(t)\frac{y^2}{2} + \Theta(t),$$

Additional shear stresses

$$\tau_1(x,t) = x\tilde{\tau}(t),\ \ \tau_2(y,t) = y\tilde{\tau}(t)$$

$A(t), B(t), \Theta(t), \tilde{\tau}(t)$ **is the arbitrary functions.**

Numerical algorithm

Numerical algorithm for determining functions Z(t), f(x,t), g(x,t)

Two-step predictor-corrector algorithm

$$Z^{k+1} = Z^{k-1} - 2\Delta t\int_0^{Z^k} f^k(z)\,dz, \quad \overline{Z}^{k+1} = Z^k - \Delta t\left[\int_0^{Z^{k+1}} f^{k+1}(z)\,dz + \int_0^{Z^k} f^k(z)\,dz\right].$$

Three-step predictor-corrector algorithm

$$q^{k+1/4} = q^k + 0.5\Delta t\left[-\Lambda_1 q^{k+1/4} + \Phi^k\right], \quad q^{k+1/2} = q^{k+1/4} + 0.5\Delta t\left[-\Lambda_2 q^{k+1/2}\right],$$

$$q^{k+1} = q^k + \Delta t\left[-\Lambda q^{k+1/2} + \Phi^k\right], \ \Lambda = \Lambda_1 + \Lambda_2, \ \Lambda_1 = -\frac{1}{Re}\frac{\partial^2}{\partial z^2}, \ \Lambda_2 = -F\frac{\partial}{\partial z}.$$

q^{k+1} is the values of the functions f and g at the grid nodes

Interpolation formulas are used to recalculate the values of functions *f*, *g* and T in the nodes of the new grid on the new time layer.

Pukhnacheva T.P., 2000

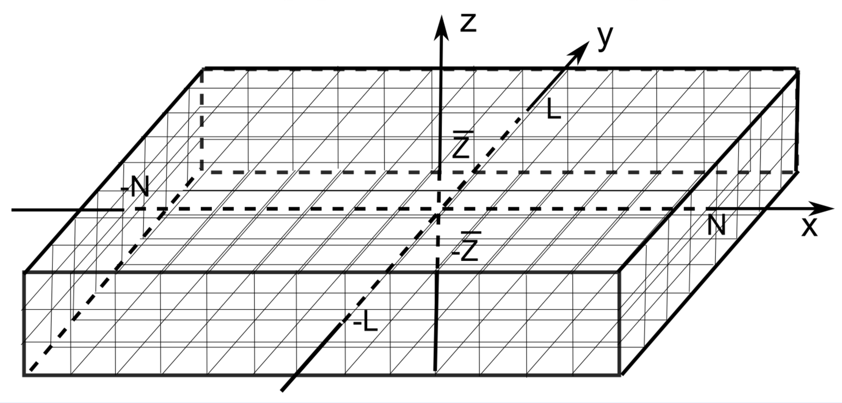
Numerical modeling of the temperature distribution in the layer

Stabilizing correction method

$$\frac{T^{k+1/3} - T^k}{\Delta t} = K_1T^{k+1/3} + K_2T^k + K_3T^k + S^k,$$

$$\frac{T^{k+2/3} - T^{k+1/3}}{\Delta t} = K_2\left(T^{k+2/3} - T^k\right), \quad \frac{T^{k+1} - T^{k+2/3}}{\Delta t} = K_3\left(T^{k+1} - T^k\right).$$

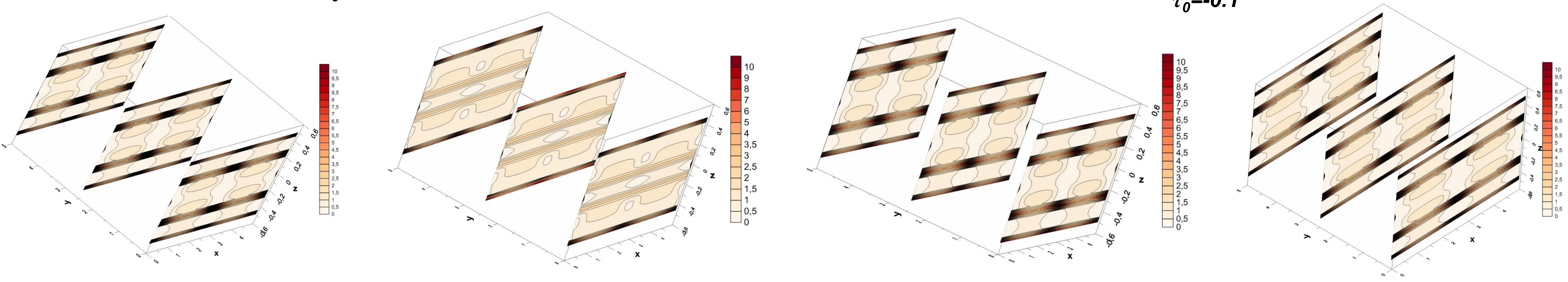
$$K_1 \approx \frac{1}{\mathrm{Re}}\frac{\partial^2}{\partial z^2}, \quad K_2 \approx \frac{1}{\mathrm{Re}}\frac{\partial^2}{\partial x^2}, \quad K_3 \approx \frac{1}{\mathrm{Re}}\frac{\partial^2}{\partial y^2}, \quad S \approx -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - w\frac{\partial T}{\partial z}.$$



Conditions T_{xx}=0 and T_{yy}=0 are used as "soft" conditions on the "vertical" ends of the study area.

Numerical results: temperature distribution in the free liquid layer

τ_0=0.1 τ_0=-0.1



t=0.05 t=0.1 t=0.05 t=0.1

Type of functions that determine the temperature regime and additional tangential stresses at the free boundaries: $A(t) = A_0te^t, \quad B(t) = B_0te^t, \quad \tilde{\tau}(t) = A_\tau\tau_0te^t. \quad A_0=B_0=-0.1.$

Results:

1. Analytical and numerical modeling of the heat transfer process in a layer of a viscous incompressible fluid with free boundaries is carried out based on exact solutions of the Navier-Stokes equations in the three-dimensional case.
2. A numerical algorithm for solving the problem of free-layer deformation under the influence of thermocapillary forces and additional tangential stresses is constracted.
3. The three-dimensional temperature distribution problem in a parallelepiped with moving boundaries is solved numerically.
4. The effect of additional tangential stresses on the dynamics of the layer and heat transfer processes is investigated.

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