THE INFLUENCE OF TANGENTIAL VIBRATIONS ON THE LONG-WAVE MARANGONI CONVECTION IN A THIN FILM ON A SUBSTRATE

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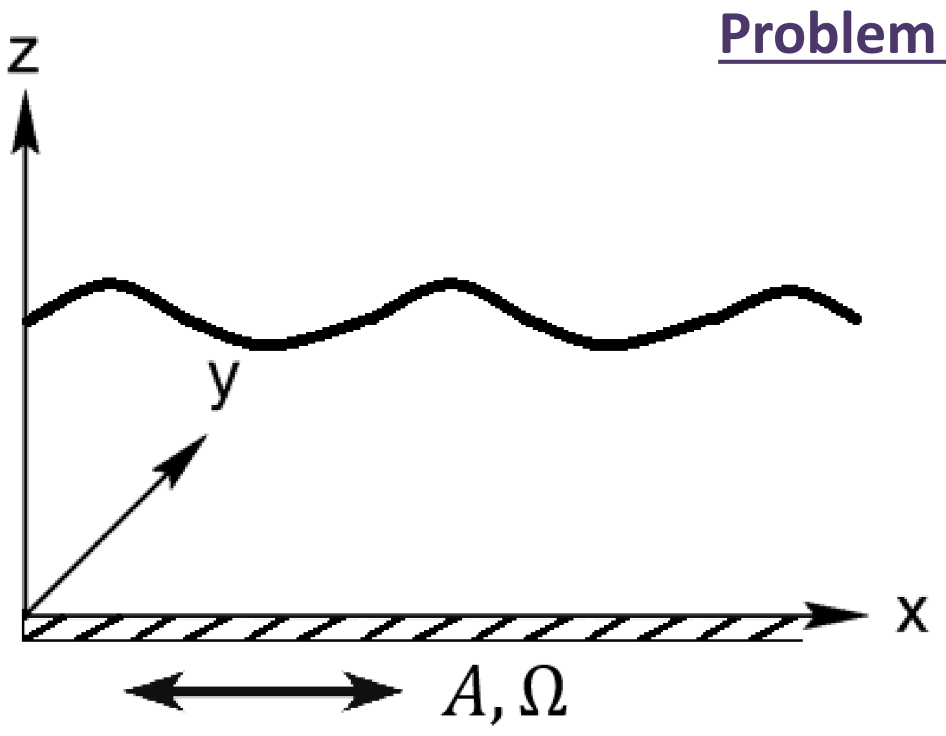
Problem formulation



$$div\,\boldsymbol{v} = 0$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\nabla\boldsymbol{v} = -\frac{1}{\rho}\nabla p + \nu\Delta\boldsymbol{v} - g\boldsymbol{k}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{v}\nabla T = \chi\Delta T$$

г.у

$z = 0$:

$$v_z = v_y = 0, v_x = A\Omega\cos(\Omega t)$$

$z = h$:

$$\frac{\partial h}{\partial t} + \boldsymbol{v}\nabla h = v_z, \qquad \frac{\partial T}{\partial z} = 1,$$

$$(p + \varphi - \sigma K)\boldsymbol{n} = \bar{\eta}\boldsymbol{n},$$

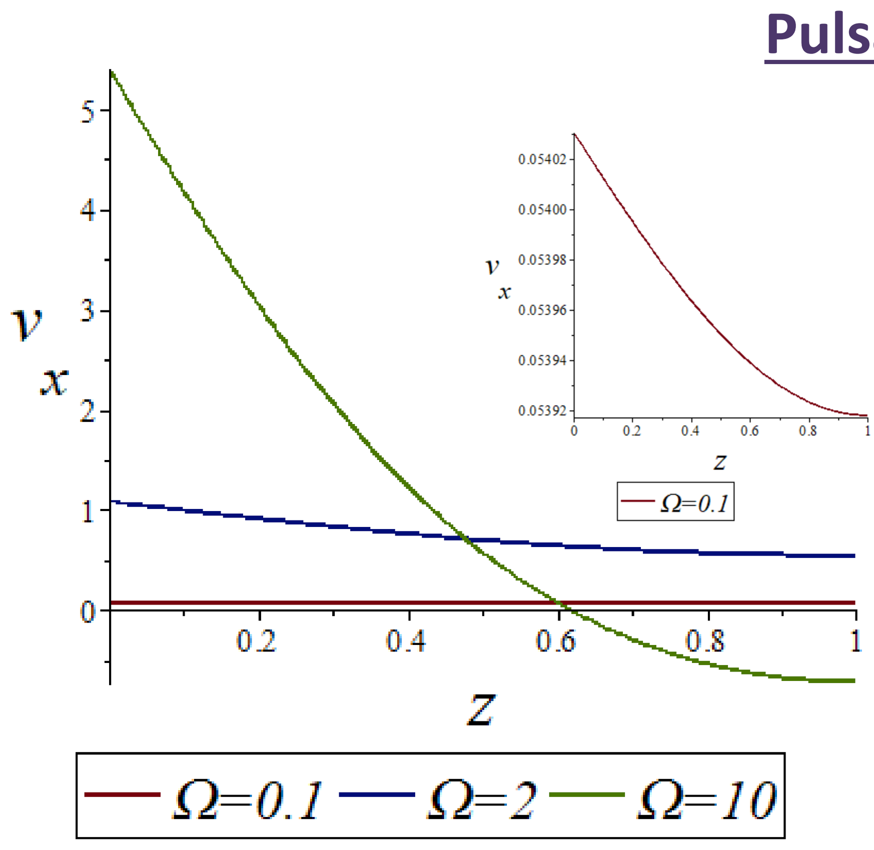
$$\boldsymbol{\tau}\bar{\eta}\boldsymbol{n} = -\frac{\partial\sigma}{\partial T}\boldsymbol{\tau}\nabla T, \qquad \boldsymbol{n}\nabla T = -\alpha T$$

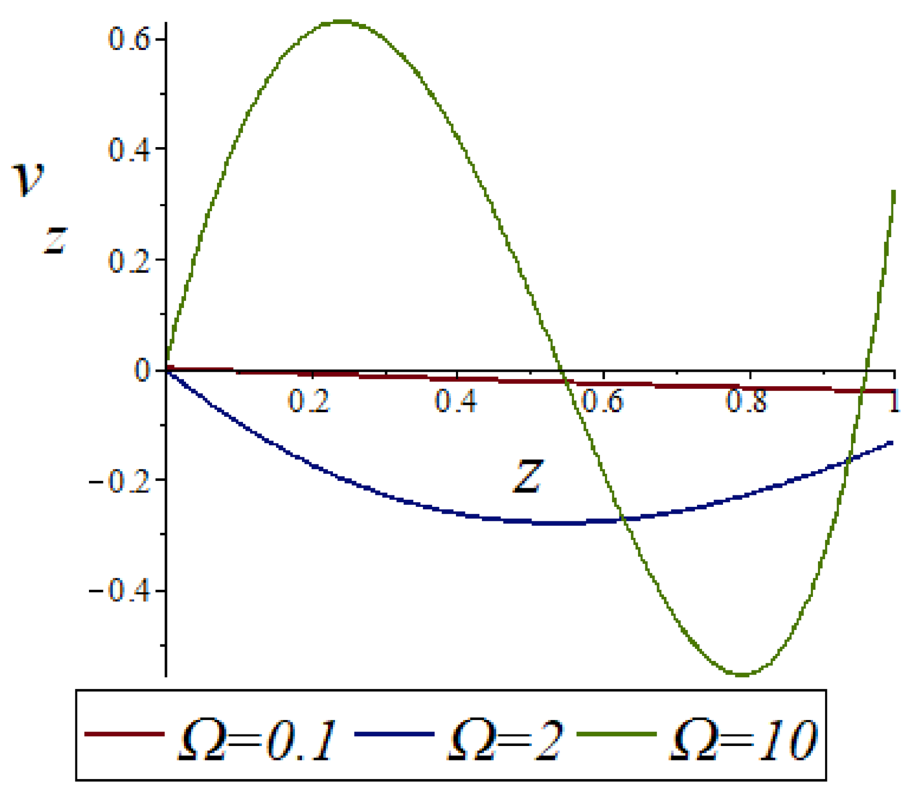
Dimensionless variables

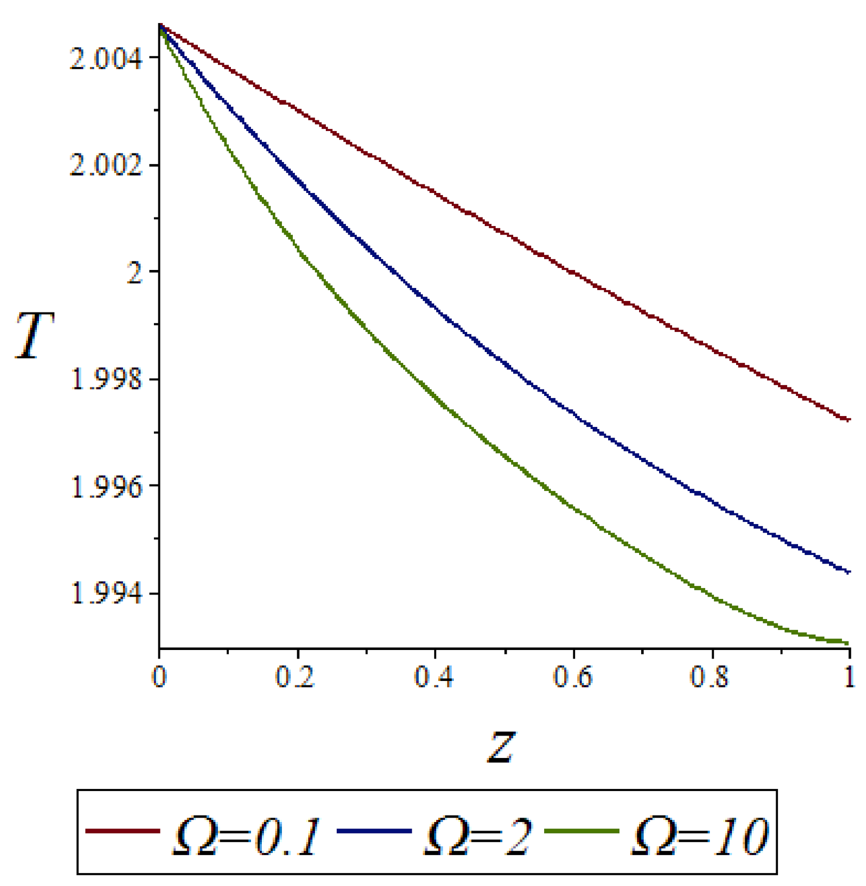
$$[H] = h_0, [t] = \frac{h_0^2}{\nu}, [v] = \frac{\nu}{h_0},$$

$$[p] = \frac{\rho\nu^2}{h_0^2}, [\frac{\partial T}{\partial z}] = \frac{T_0}{h_0}$$

Pulsatile flow







$$\widetilde{p} = \widetilde{v_y} = 0.$$

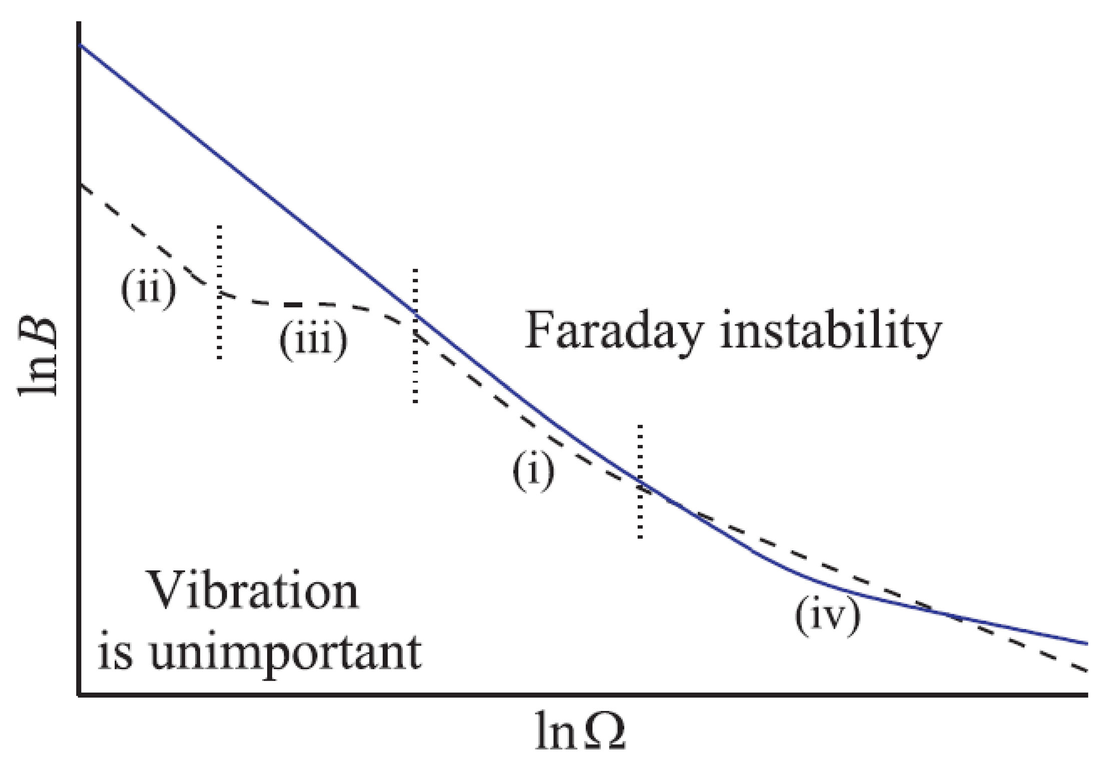
$$\tilde{U} = A\Omega\,\mathrm{Re}\left[v_x(X,Z)e^{i\Omega\tau}\right],$$

$$\tilde{W} = A\Omega\,\mathrm{Re}\left[v_z(X,Z)e^{i\Omega\tau}\right],$$

$$\tilde{h} = A\,\mathrm{Re}\left[H(X)e^{i\Omega\tau}\right],$$

$$\tilde{T} = A\Omega\,\mathrm{Re}\left[T(X,Z)e^{i\Omega\tau}\right]$$

Vertical vibrations



(i) Average approximation

$\Omega \sim O(1)$, $b \sim O(\varepsilon^{-1})$,

(ii) Ultralow frequency

$\Omega \sim O(\varepsilon^2)$, $b\Omega^2 \sim O(1)$,

(iii) Low frequency

$\Omega \sim O(\varepsilon^\beta)$, $b \sim O(\varepsilon^{-1-3\beta/2})$,

$0 \le \beta \le 2$

(iv) High frequency

U. Thiele, J. M. Vega, and E. Knobloch, J. Fluid Mech. 546, 61 (2006).

S. Shklyaev, A. A. Alabuzhev, and M. Khenner, Phys. Rev. E. 2015. V. 92, 013019.

Method of solution

Average and pulsatile flows. Scalenig

Медленное время: $t = \varepsilon^2 t, \varepsilon = \frac{H}{L} \ll 1.$

Быстрое пульсационное время: $\tau = \Omega t.$

$Z = z, X = \varepsilon x, Y = \varepsilon y.$

$\boldsymbol{v} = <\boldsymbol{v}> + \widetilde{\boldsymbol{v}}, v_z = \varepsilon^2 <v_z> + \varepsilon\widetilde{v_z}$

$<\boldsymbol{v}>$ - осредненная, $\widetilde{\boldsymbol{v}}$ -пульсационная скорость

$v_x = \varepsilon <v_x> + \widetilde{v_x}, v_y = \varepsilon <v_y> + \widetilde{v_y},$

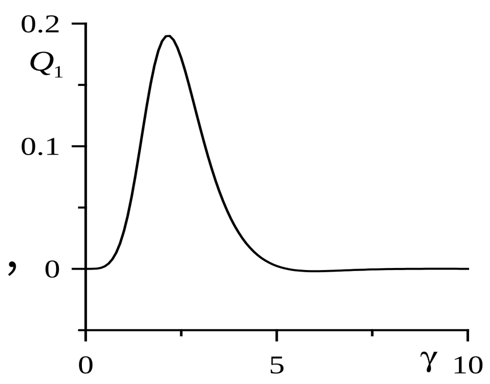
$p = <p> + \frac{1}{\varepsilon}\tilde{p}, h = \varepsilon <h> + \tilde{h}, T = <T> + \tilde{T}$

Amplitude equation (non-isotherm case)

$$h_t = \frac{1}{3}\nabla\cdot\left(h^3\nabla\Pi\right) - \frac{1}{2}A^2\Omega^2\left(Q_1 h^2 h_X\right)_X$$

$$\Pi = -\varphi(h) + Gh - C\Delta h, \quad \gamma = h\sqrt{2\Omega},$$

$$Q_1 = 3\frac{2\sin\gamma\,\mathrm{sh}\,\gamma - \gamma(\cos\gamma\,\mathrm{sh}\,\gamma + \mathrm{ch}\,\gamma\sin\gamma)}{\gamma^2(\mathrm{ch}\,\gamma + \cos\gamma)^2},$$



Small perturbations

$$h = 1 + \xi(X,Y,T), V = \frac{A^2\Omega^2}{2a}, \gamma_0 = \sqrt{2\Omega}$$

$$\xi_T = \Delta\left[(G_0 - 1)\xi - \Delta\xi\right] - VQ_1(\gamma_0)\xi_{XX},$$

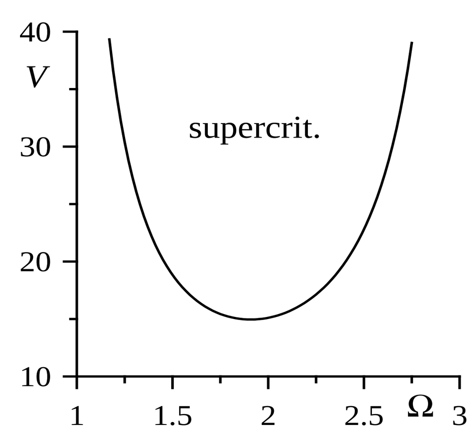
$$\xi = \hat{\xi}\exp(-\lambda T + ik_X X + ik_Y Y),$$

$$\lambda = k^2\left[G_0 - 1 + k^2\right] - VQ_1(\gamma_0)k_x^2.$$

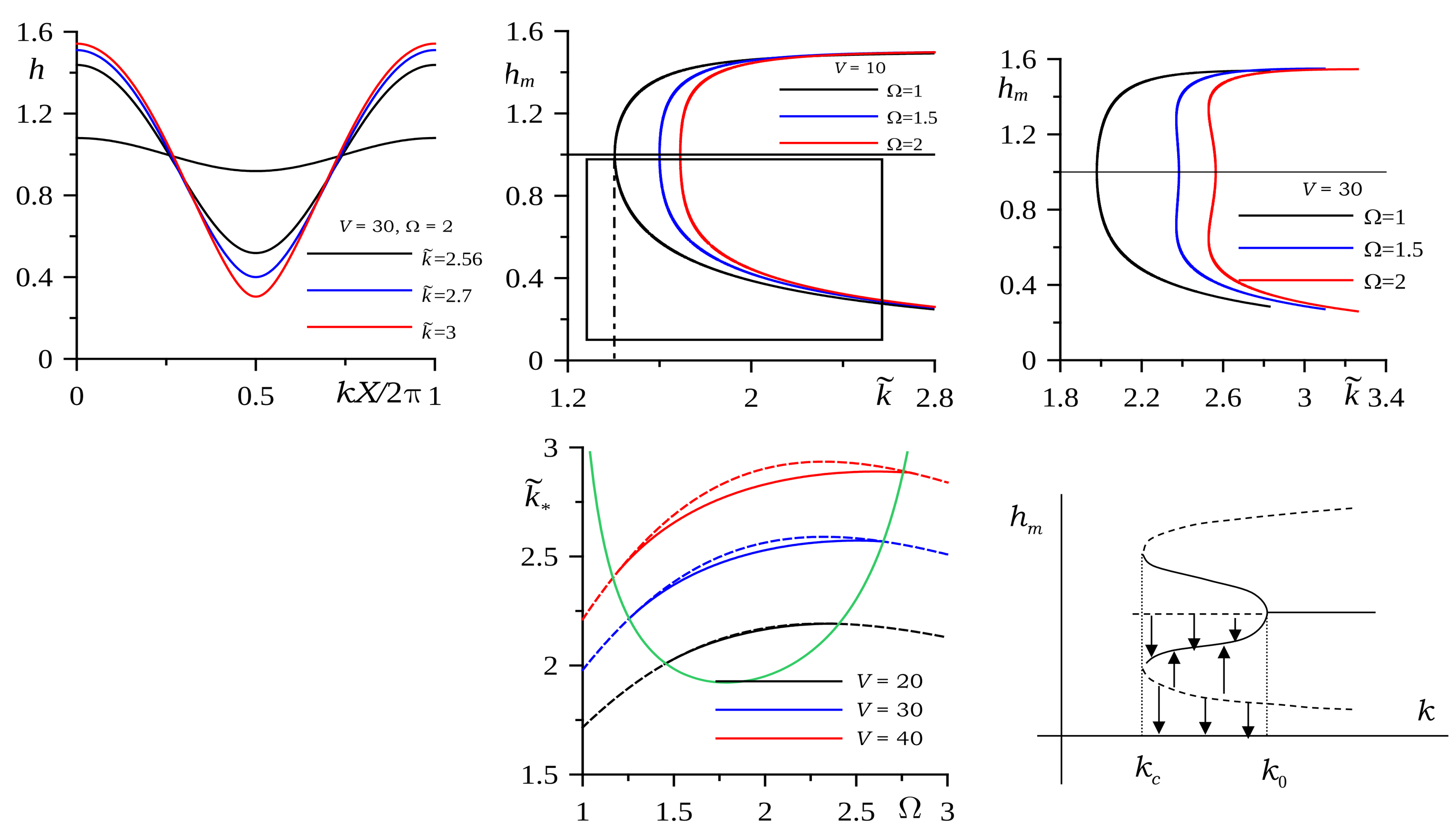
Weakly nonlinear analysis

Стационарные решения малой амплитуды и малое отличие волнового числа от критического

$$h = 1 + \delta h_1 + \delta^2 h_2 + \ldots, \quad \tilde{k} = \tilde{k}_c + \delta^2\tilde{k}_2 + \ldots$$



Unstable stationary solution



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