Lavrentyev Institute of Hydrodynamics SB RAS
Steklov Mathematical Institute RAS
Institute for System Dynamics and Control Theory SB RAS
Novosibirsk State University

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ABSTRACTS

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Contents

Pulse solutions of some hydrodynamical problems in unbounded domains

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We consider several hydrodynamic problems in unbounded domains where in the vicinity of the instability threshold the dynamics is governed by the generalized Cahn-Hilliard equation. For time independent solutions of this equation we recover Bogdanov-Takens bifurcation without parameter in the 3-dimensional reversible system with a line of equilibria. This line of equilibria is neither induced by symmetries, nor by first integrals. At isolated points, normal hyperbolicity of the line fails due to a transverse double eigenvalue zero. In case of bi-reversible problem the complete set $\mathcal{B}$ of all small bounded solutions consists of periodic profiles, homoclinic pulses and a heteroclinic front-back pair (Asymptot. Anal. 60(3,4) (2008), 185–211). Later the small perturbation of the problem where only one symmetry is left was studied. Then $\mathcal{B}$ consist entirely of trivial equilibria and multipulse heteroclinic pairs (Asymptotic Analysis, Volume 72, Number 1-2 , 2011, pp. 31-76). Our aim is to discuss hydrodynamic problems, where the reversibility breaking perturbation can't be considered as small. We obtain the existence of a pair of heteroclinic solutions and partial results on their stability.

Boundary control problems in hydrodynamics and heat and mass transfer

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Much attention has recently been given to statement and investigation of new problems for models of continuum mechanics. Control problems for models of hydrodynamics, heat and mass transfer and magnetic hydrodynamics are examples of these problems. The interest to control problems is connected with a variety of important applications in science and engineering such as the crystal growth process, aerodynamic drag reduction, suppression of turbulence and flow separation. In these problems unknown densities of boundary or distributed sources are recovered from additional information on the solution to the boundary value problem. Control and inverse problems for these models are analyzed by applying a unified approach based on the constrained optimization theory in Hilbert or Banach spaces (see [1, 2]).

Our goal is the study of boundary control problems for the Navier-Stokes and Boussinesq equations describing the flow of the viscous incompressible heat conducting fluid in a bounded domain. These problems consist in minimization of certain cost functionals depending on the state and controls. The existence of an optimal solution is based on a priori estimates and standard techniques. Optimality systems describing the first-order necessary optimality conditions are obtained, and, by analysis of their properties, conditions ensuring the uniqueness and stability of solutions are established.

Two algorithms for the numerical solution of control problems are proposed. The first of them is based on application of the gradient method. This numerical algorithm reduces the solution of the control problem to the solution of direct and adjoint problems at each iterative step. The second algorithm is based on solution of the optimality system applying Newton's method. The influence of the initial guess and parameters of the algorithm on the speed of convergence and accuracy of the obtained numerical solution is investigated. The open source software freeFEM++ (www.freefem.org) is used for the discretization of boundary-value problems by the finite element method. Some results of numerical experiments can be found in [1, 2, 3].

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About incompressible Euler limit of solutions of Navier-Stokes and Boltzmann equation in the presence of boundary effects

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It is well known that in any dimension (even in 2d) the limit (when the Reynold number goes to infinity) is in presence of boundary a challenging open problem...

Results are simpler when the fluid satisfies a Navier boundary condition and the problem is completely open when the fluid satisfies for finite Reynold number a Dirichlet boundary condition. The only general (always valid) mathematical result being a classical theorem of Tosio Kato.

In the incompressible finite Reynold number limit the solution of the Boltzmann equation (with boundary and accomodation effect) converges to a Leray solution of the Navier Stokes equation with a Navier Boundary condition which depends on the accomodation coefficient (Kazuo Aoki...Nader Masmoudi and Laure Saint Raymond)

On the other hand it has been observed by several researchers from Yoshio Sone to Laure Saint Raymond that with convenient scalings (infinite Reynolds number) and in the absence of boundary the Boltzmann equation leads to the incompressible Euler equation.

Hence we try to adapt to this limit, in presence of boundary with accomodation, what is known or conjectured at the level of the Navier Stokes limit.

On some problems of convection in Lake Baikal

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This paper features a review of investigations into the stability of equilibrium states and convective flow of fluid with complex rheology, which were performed with regard to Lake Baikal in ICM SB RAS.

To study waters deep circulation mechanism [1], a number of problems of hydrodynamic stability theory was considered. In the framework of Oberbeck–Boussinesq model the peculiarities of thermal and dense stratification of aqua mass were taken into account:
– state equations of weakly compressible media were used (it allows for water compressibility effect on large depths);
– a general heat source energy function was introduced into the energy equation (it describes the absorption of solar radiation and defines the thermal stratification of fluid in the ice-free period and also through the ice cover).

The stability problems of equilibrium states of the viscous heat-conducting weakly compressible fluid layer and two-layer systems were solved. The task on penetrative convection beginning in temperature fluctuation conditions on free surface was considered.

The solution describing convective flow in layer with rigid boundaries (upper wall is ice) was obtained and its stability was investigated.

The instability domains and parameters of disturbing influences were defined in all the problems.

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Mathematical theory of nonlinear resonance for abstract hyperbolic equations

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The typical problem of perturbation theory for hyperbolic equations has the form

$$\ddot{u} = -A^2 u + \varepsilon F(\omega t, u),$$

where $u(t)$ is a function with values in a Hilbert space, A is a linear self-adjoint unbounded operator, ε is a small parameter, $F(\tau, u)$ is a continuous perturbation periodic or almost periodic on τ, $F(\tau, 0) \equiv 0$. If $\varepsilon = 0$ then trivial solution to this equation is stable. We are interested in parametric resonance i.e. the loss of stability for all $\varepsilon \neq 0$ small enough if a frequency ω is closed to some critical values. It is well known that critical frequencies for linear equations are defined by spectral properties of the operators A and F, and what's more, under resonance conditions the amplitudes of oscillations increase in time exponentially. In nonlinear case the critical friquencies depend also on amplitudes. Being satisfied in the initial moment, the resonance conditions may become false when amplitudes growth. Therefore instead of unbounded increase we observe "pulsations" of amplitudes, the theory of which has been elaborated only for Hamiltonian systems in finite dimensional space [1, 2].

We present the corresponding theory in the infinite dimensional space for quadratic perturbations of the form $B(\omega t)u + Q(u, u)$, where $B(\tau)$ is a linear bounded operator periodic on τ, and $Q(u, v)$ is bilinear Hermitian form. It is prooved that on the time intervals of the order ε^{-1} the amplitude of oscillations is approximately described by a finite dimensional dynamic system. This system may be found by the Krylov — Bogolubov averadging method, which must be specially modified to get over the known "small divisor problem". As a result it becomes possible to explaine the qualitative picture of pulsations under nonlinear resonance and to elaborate a numerical algorithm of approximate calculation of solutions to initial hyperbolic equation.

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Weakly nonlinear surface waves

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In scale invariant boundary value problems, linear surface waves of finite energy like the Rayleigh waves in elasticity are associated with modulated waves in the weakly nonlinear regime, which are governed by an amplitude equation that is a nonlocal generalization of Burgers' equation. In this talk, we will make the connection between the structure and stability properties of the amplitude equation and those of the original, fully nonlinear problem. We will consider two types of problems: first-order hyperbolic systems of PDEs and higher-order Hamiltonian PDEs, the prototypes of which are the Euler equations of gaz dynamics and the equations of elastodynamics.

Well posedness of some multi-fluid systems

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In this talk, we will present some recent mathematical features around multi-fluid models. Such systems may be encountoured for instance to model internal waves, violent aerated flows, oil-and-gas mixtures. Depending on the context, the models used for simulation may greatly differ. However averaged models share the same structure. Here, we address the question whether available mathematical results in the case of a single fluid may be extended to multi-phase models for instance those available for the compressible barotropic equations for single flow.

Stability estimates of solutions of control problems for stationary equations of magnetohydrodynamics

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In recent years, much attention has been given to optimal control problems for flows of viscous electrically conducting fluids. The study of these problems was motivated by the necessity of the most effective control mechanisms for hydrodynamic processes in such fluids. A rigorous theoretical study of these problems can be found, for example, in [1-4].

Along with optimal control problems, an important role is played by identification problems for MHD models. In the latter problems, the unknown coefficients involved in the differential equations or in the boundary conditions for the model in question are determined from additional data on the solution. Note that identification problems can be reduced to optimization problems with a suitable choice of the minimized cost functional. In [4] this approach was used to analyze the solvability, uniqueness and stability of solutions to identification problems for models of magnetohydrodynamics of viscous incompressible fluid.

In this paper identification problems for the stationary magnetohydrodynamic (MHD) model governing a flow of a viscous electrically-conducting fluid are stated and analyzed. This model consists of the Navier-Stokes equations, the generalized Ohm law, and the stationary Maxwell equations without displacement currents, considered under Dirichlet boundary condition for the velocity and mixed boundary conditions for the electromagnetic field. The solvability of the problem is proved, an optimality system is derived, and sufficient conditions on the initial data are established that ensure the uniqueness and stability of the solution.

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Low regularity water-waves

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I will explain how to reduce the water-wave system to a wave-type equation. For gravity water waves, the linearized equation reads

$$\partial_t u + i|D_x|^{1/2} u = 0 \quad (x \in R^d, d = 1 \text{ or } 2).$$

We reduce the nonlinear water-wave system to an equation of the form

$$\partial_t u + T_V \cdot \nabla u + iT_\gamma u = f,$$

where T_V is a paraproduct and T_γ is a paradifferential operator. This reduction allows to study dispersive properties of gravity waves and also to solve the Cauchy problem for 3D-waves in an horizontal channel.

Analytical solutions of the multilayered rotating shallow water equations

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The multilayered rotating shallow water (MRSW) theory is a widely-used approximation for atmospheric and oceanic motions. In the polar coordinates the MRSW equations have the following form

$$\frac{\partial U_i}{\partial t} + U_i \frac{\partial U_i}{\partial r} + \frac{V_i}{r}\frac{\partial U_i}{\partial \theta} - \frac{V_i^2}{r} - fV_i + \\ + g\frac{\partial}{\partial r}\Big(Z - \frac{f^2 r^2}{8g} + \sum_{k=1}^{i} h_k + \frac{1}{\rho_i}\sum_{k=i+1}^{N} \rho_k h_k\Big) = 0,$$

$$\frac{\partial V_i}{\partial t} + U_i \frac{\partial V_i}{\partial r} + \frac{V_i}{r}\frac{\partial V_i}{\partial \theta} + \frac{U_i V_i}{r} + fU_i + \\ + \frac{g}{r}\frac{\partial}{\partial \theta}\Big(Z + \sum_{k=1}^{i} h_k + \frac{1}{\rho_i}\sum_{k=i+1}^{N} \rho_k h_k\Big) = 0,$$

$$\frac{\partial h_i}{\partial t} + \frac{1}{r}\frac{\partial (rU_i h_i)}{\partial r} + \frac{1}{r}\frac{\partial (V_i h_i)}{\partial \theta} = 0; \qquad Z = \frac{\kappa r^2}{2}, \quad \Big(\kappa \geq \frac{f^2}{4g}\Big) \qquad (1)$$

Here (U_i, V_i) is the fluid velocity, h_i is the depth of the i-th liquid layer, Z is the height of the base of the liquid above some fixed level, f is the constant Coriolis parameter, g is the constant gravity acceleration. Our attention is restricted to basin geometries corresponding to a circular paraboloid.

We have studied classes of exact solutions of the equations using group analysis. This analysis, in particular, allows one to show that the MRSW equations (1) are related with the multilayered shallow water (MSW) model (equations (1) with $f = 0$, $Z = 0$) through the following change of variables:

$$t' = \frac{2}{\omega}\tan\frac{\omega t}{2}, \quad r' = \Big(\cos\frac{\omega t}{2}\Big)^{-1} r, \quad \theta' = \frac{ft}{2} + \theta; \qquad \big(\omega = 2\sqrt{g\kappa}\big)$$

$$U_i' = U_i \cos\frac{\omega t}{2} + \frac{\omega r}{2}\sin\frac{\omega t}{2}, \quad V_i' = \Big(V_i + \frac{fr}{2}\Big)\cos\frac{\omega t}{2}, \quad h_i' = h_i \cos^2\frac{\omega t}{2}.$$

This transformation allows one to construct and study solutions of the MRSW equations using solutions of the MSW model and vice versa.

Using admissible symmetries we have generated new time-periodic exact solutions of the MRSW equations. These solutions describe fluid flow with quasi-closed (ergodic) particle trajectories and may be interpreted as pulsation of stratified liquid volume under the influence of gravity and Coriolis forces. The obtained results are development and generalization of the work [1] for the case of a stratified fluid over an uneven bottom.

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New algoritm of group classification of systems of differential equations

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Differential equations of mechanics and mathematical physics often contain include parameters and functions defined with the help of experiments. It's parameters and functions are an arbitrary element of these equations. Group classification of it's equations permits to obtain values and forms of these parameters and functions guarantee existence of additional symmetries of considered mathematical models. Models admitting additional symmetries as a rule are the most perspective for mathematical research.

We offer a new algorithm of group classification of system of differential equations. Efficiency and preferences of it's algorithm are illustrated with a help of gas dynamics equations and equations of nonlinear longitudinal oscillations of visco-elastic rod in Kelvin's model. A new algorithm has following possess an advantages over classical algorithm [1]: 1) there is no necessity to solve difficult problems connected with classifying equations; 2) volume of calculations is essentially reduced; 3) group of equivalences is found at once for every particular arbitrary element.

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On solution non-uniqueness in the nonlinear elasticity theory

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The simultaneous effects of dissipation and dispersion on nonlinear wave behavior in elastic media are considered when the effects are small and manifested only in narrow high-gradient regions. If one constructs solutions of self-similar problems in "hyperbolic" approximation using Riemann's waves and admissible discontinuities (i.e., discontinuities with structures) one obtains many solutions the number of which unlimitedly grows with growing the relative influence of dispersion (as compared to dissipation) in discontinuity structures.

The numerical analysis (based on PDE with dispersion and dissipation) of nonself-similar problems with self-similar asymptotics is performed to determine which of self-similar solutions is an asymptotic form for the nonself-similar solution.

Global solvability of a problem governing the motion of two incompressible capillary fluids

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We deal with the motion of two incompressible fluids in a container, one of which is inside another. We take surface tension into account. The inner liquid occupies the domain $\Omega_t^+ \subset \mathbb{R}^3$, and the outer one is in $\Omega_t^- \subset \mathbb{R}^3 \setminus \overline{\Omega_t^+}$. They are separated by the unknown closed interface $\Gamma_t \equiv \partial\Omega_t^+$, at initial moment $t = 0$ Γ_0 being given. The outer boundary

$S \equiv \partial(\Omega_t^+ \cup \Gamma_t \cup \Omega_t^-)$ is a given surface. We assume $S \cap \Gamma_0 = \emptyset$. It is necessary to find Γ_t, as well as the velocity vector field $\mathbf{v} = (v_1, v_2, v_3)$ and the pressure function p of both fluids satisfying the initial–boundary value problem for the Navier–Stokes system

$$\frac{\partial \mathbf{v}}{\partial t} - \nu^\pm \nabla^2 \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{1}{\rho^\pm} \nabla p = 0, \quad \nabla \cdot \mathbf{v} = 0 \quad \text{in } \Omega_t^\pm,\ t > 0,$$

$$\mathbf{v}|_{t=0} = \mathbf{v}_0, \quad [\mathbf{v}]|_{\Gamma_t} = 0, \quad [\mathbb{T}\mathbf{n}]|_{\Gamma_t} = \sigma H \mathbf{n}, \quad \mathbf{v}|_S = 0,$$

where $\nu^\pm, \rho^\pm$ are the step–functions of fluid viscosities and densities, respectively, $\mathbf{v}_0$ is the initial velocity distribution, $\mathbb{T}$ is the stress tensor with the elements

$$\{\mathbb{T}(\mathbf{v}, p)\}_{ik} = -p\delta_k^i + \mu^\pm(\partial v_i/\partial x_k + \partial v_k/\partial x_i),\ i, k = 1, 2, 3;$$

$\mu^\pm = \nu^\pm \rho^\pm$ are the step–functions of dynamical viscosities, $\mathbf{n}$ is the outward normal to Ω_t^+; $[\mathbf{w}]|_{\Gamma_t}$ is the jump of the vector $\mathbf{w}$ across Γ_t from Ω_t^+ to Ω_t^-; $\sigma > 0$ is the coefficient of surface tension, H is twice the mean curvature of Γ_t ($H < 0$ at the points where Γ_t is convex towards Ω_t^-).

Moreover, to exclude mass transportation through Γ_t, we assume that the liquid particles do not leave Γ_t. It means that the velocity of interface motion in the normal direction coincides with the normal part of fluid velocity: $V_{\mathbf{n}} = \mathbf{v} \cdot \mathbf{n}|_{\Gamma_t}$.

We prove that this problem is uniquely solvable in an infinite time interval $t > 0$ provided the initial velocity of the liquids is small and the initial configuration of Ω_0^+ is close to a ball $\{|x| \le R_0\}$. Moreover, we show that the velocity decays exponentially as $t \to \infty$ and the interface Γ_t tends to the sphere $\{|x - h| = R_0\}$ with a certain small $h \in \mathbb{R}^3$. The proof is based on the exponential estimate of a generalized energy and on a local existence theorem of the problem in anisotropic Hölder spaces [1]. We follow to the scheme developed by one of the authors for proving global solvability of a problem governing the motion of one incompressible capillary fluid bounded by a free surface [2].

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Simple asymptotics for waves and vortices with small amplitudes on the shallow water created by localized sources

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We discuss the new asymptotic method for construction of asymptotic solutions to Cauchy problems with localized initial data (perturbations) and right hand sides for the 2-D linearized Shallow Water Equation over nonuniform bottom, including the case when the basin depth tends to zero on a certain smooth curve. We suggest new asymptotic representation for solutions of these problems which is the generalization of the Maslov canonical operator. It is based also on a simple relationship between fast decaying and fast oscillating solutions and on boundary layer ideas. Our main result is the explicit formulas which establish the connection between initial localized perturbations and wave profiles near the wave fronts including the neighborhood of backtracking (focal or turning) and self intersection points. Also we show that in the case when the original system possesses the vortical solutions, the solitary vortices correspond to the the focal points. We point out the wide class of initial perturbation resulting to asymptotic formulas expressed via elementary functions and based on "creation operators". We discuss the influence of such topological characteristics like the Maslov and Morse indices to metamorphosis of the profiles of localized solutions after crossing of the focal points and reflecting from the shore line as well as the appearance of the Hilbert transform in asymptotic formulas etc. Finally we discuss the influence of nonlinear effects. We apply our constructions to the problem of a propagation of tsunami waves in the frame of so-called “piston model” and also to the problem of propagation of mesoscale vortices (typhoons and hurricanes) in the atmosphere.

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Curvilinear coordinates for fluids with frozen-in vector fields

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Some mathematical models of fluid mechanics involve vector fields that are frozen into the fluid flow. These are the vortex field in the model of ideal fluid, magnetic field in ideal MHD, generalized vorticity for bubbly liquid and dispersive shallow water and some others. Integral curves of such fields are transported by the flow, and also influence on the flow. Description of the interaction of these two actions is an essentially nonlinear and three-dimensional problem that is difficult to solve analytically and numerically.

In present talk we use a curvilinear coordinate system for which streamlines and integral curves of the frozen-in fields serve as two families of coordinate curves. This approach allows partial integration of equations of the model, and gives opportunity for construction of special classes of solutions. We give examples of exact solutions for ideal MHD equations that describe flows with total pressure constant over the whole space occupied by the flow [1, 2]. The interesting feature of obtained solutions is nontrivial topology of magnetic tubes which have shape of nested tori or knots.

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Convection under low gravity: models, analytical and numerical investigations

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Modeling of the convective processes caused by impact of various forces on the fluid and gas media is rather important nowadays. The increased interest to these problems is determined by preparation of the experiments on the International Space Station. Some of them are the experiments to investigate the convective flows of the fluids with a thermocapillary interface between liquid and gas phases. Mathematical modeling of the various convective processes should be carried out: formulation of the problem statements, formulation of the conditions at the evaporative interface, construction of the exact solutions in the canonical domains, analytical and numerical investigation of the simplified problems.

The alternative mathematical models of convective fluid flows (the microconvection model of isothermally incompressible liquid, the model of convection of weakly compressible liquid) and the classical Oberbeck-Boussinesq model are applicable to investigation of many problems of convection: convection under low gravity, in small scales and at fast changes of the boundary thermal regimes [1]. Making use a special and physically correct form of the state equation, the model of microconvection can be reformulated in the terms of modified solenoidal velocity. It leads to a demand that the total flux through the boundary of a closed cavity should be equal to zero at each time moment. The model of convection of weakly compressible liquid is free of this crucial condition. Principal issues relating to well/ill posed initial boundary value problems for the equations of convection are considered. The analytical results concerning the correctness of the non-standard problems for the microconvection equations and of the problems for the equations of convection of weakly compressible liquid are presented. For the convection equations of weakly compressible liquid the initial boundary value problem with general temperature condition on the boundary is studied. The local theorem of existence of a smooth solution in the classes of the Hoelder functions is proved.

The examples of the invariant solutions of the alternative models of convection in an infinite strip are presented.

In the problem of convection of two immiscible fluids with an interface the exact solutions are constructed in the three-dimensional case. The flows in a channel with a rectangular cross-section are caused by longitudinal temperature gradient.

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Quasipatterns in a parametrically forced horizontal fluid film

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We consider a horizontal fluid film, vertically harmonically shacked (frequency forcing 1, only one harmonic). In addition to the amplitude Γ of the forcing, other parameters of the system are the Reynods number R, the Froude number Fr, and the Bond number Bo. We give the conditions under which the basic time periodic flow losses its stability, via time periodic perturbations of period 4π or 2π with the same critical value Γ_c of the forcing. These instabilities lead to spatial wave numbers k and k', corresponding to critical circles in the Fourier plane (due to rotational invariance of the system).

Now consider the cases when $k'/k = 2cos(n\pi/q)$ with n and q integers, and denote by k_j (resp. k'_j) the wave vectors of length k (resp. k'), making the angle $(j-1)\pi/q$ with the x axis, then this condition corresponds to $k_j + k_{j+2n} = k'_{j+n}$, for $j = 1, \dots 2q$ and these cases occur for values of R and $Bo(Fr)^2$ depending only on n/q.

Quasipatterns correspond to the cases $q \geq 4$.

Searching for solutions invariant under rotations of angle $2\pi/q$, we reduce formally the problem to a system of amplitude equations. In the case where $q \geq 5$ is odd, we find 4π-time periodic quasipatterns such that a time shift of 2π is the same as rotating the pattern by π/q (this type of phenomenon is observed experimentally). We give simple conditions for their stability.

Inclined impact of smooth body onto thin liquid layer

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Inclined impact of a body is usually studied in connection with the emergency landing of aircraft onto the water. The knowledge of the body motions during the process and the hydrodynamic loads acting on the body surface are of primary interest. The process of the inclined impact can be divided into two phases. During the first phase spray jets are formed at the periphery of the contact region. During the second phase the free surface of the liquid separates from the surface of the moving body in the rear part of it. The first phase is referred to as the impact phase and the second one as planing phase.

Two-dimensional problem of inclined impact of a rigid body with smooth surface onto the thin layer of an ideal incompressible fluid is considered. The problem is coupled: liquid flow, body motion, hydrodynamic loads distributed along the contact region and the position of this contact region on the body surface should be determined simultaneously. The liquid flow during the first stage is obtained by using the approach from [1]. This approach is based on the method of matched asymptotic expansions. The flow region is subdivided into several subregions: the region beneath the entering body surface, the jet root, the spray jet, and the outer region. A complete solution is obtained by matching the solutions within these subdomains. At the second phase main attention is given to conditions at the separation point.

The coupled problem is reduced to a system of integro-differential equations. The equations are solved numerically. Displacement and rotation of the body caused by the hydrodynamic loads during both phases are investigated.

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High order well-balanced finite volume schemes for systems of balance laws

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We propose a new family of high order well-balanced schemes for hyperbolic systems of balance laws. Our construction is based on the use of two sorts of variables—conservative and equilibrium ones. The former variables make the scheme conservative, while the latter ones are used for proper well-balanced reconstruction. We apply our technique to the shallow water equations. The numerical tests confirm the well-balanced property of the schemes and high order of approximation.

Consider a hyperbolic system of balance laws

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = g(x,u), \quad (t,x) \in [0,\infty) \times [a,b], \quad u(t,x) \in \mathbb{R}^m, \qquad (1)$$

with the initial conditions

$$u(0,x) = u_0(x), \quad x \in [a,b].$$

The boundary conditions are any appropriate ones. In our tests we use periodic boundary conditions

$$u(t,a) = u(t,b), \quad t \in [0,\infty).$$

When constructing numerical schemes for hyperbolic systems of balance laws, a problem of balancing between flux gradient and right-hand side term at the discrete level arises. This balancing should be treated carefully when modeling solutions close to stationary ones. An improper discretization can originate non-physical oscillations comparable with physical disturbances of the solution. Schemes achieving that balance are called *well-balanced*.

Finite volume schemes are obtained by integration of Eq. (1) over an interval $I_j = (x_{j-1/2}, x_{j+1/2})$, $j = 1, \ldots, J$:

$$\frac{d\bar{u}_j}{dt} = \frac{1}{\Delta x}\big(F_{j-1/2} - F_{j+1/2}\big) + \langle g \rangle_j\,(t). \tag{2}$$

Here $\bar{u}_j$ is the cell average of $u(x)$ over I_j; $F_{j+1/2}$ is some intercell numerical flux: $F_{j+1/2} = F\big(u^-_{j+1/2}, u^+_{j+1/2}\big)$, where $u^\pm_{j+1/2}$ are the limiting values of u at $x_{j+1/2}$ obtained by a suitable reconstraction; $\langle g \rangle_j$ is the cell average of the source term.

A numerical scheme will be *well-balanced* if we define $u^\pm_{j+1/2}$ and $\langle g \rangle_j$ in such a way that the right-hand side of Eq. (2) vanishes at steady-state solutions.

We write our scheme (2) in *conservative* variables u. That makes the scheme conservative one. To make the scheme well-balanced we introduce so called *equilibrium* variables v. The equilibrium variables are defined as such variables which are constant at stationary solutions. We suppose that there exists a one-to-one mapping $u = U(x, v)$.

The idea is to use conservative variables u for the outer form of the scheme, while to use equilibrium variables v for internal reconstruction and computation of the intercell limits $u^\pm_{j+1/2}$ and source cell averages $\langle g \rangle_j$.

The first-order scheme is constructed in the following way. Given the cell averages $\bar{u}_j$ we define equilibrium cell averages $\bar{v}_j$ as constants which satisfy the integral equation

$$\frac{1}{\Delta x}\int_{I_j} U(x, \bar{v}_j)\,dx = \bar{u}_j. \tag{3}$$

Then using these values $\bar{v}_j$ we define intercell boundary values of conservative variables $u^\pm_{j+1/2}$ as

$$u^-_{j+1/2} = U(x_{j+1/2}, \bar{v}_j), \quad u^+_{j+1/2} = U(x_{j+1/2}, \bar{v}_{j+1}),$$

and the average of the source term as

$$\langle g \rangle_j = \frac{1}{\Delta x}\int_{I_j} g\big(x, U(x, \bar{v}_j)\big)\,dx.$$

Application of the Euler's method to semi-discrete equation (2) gives the first-order scheme

$$\bar{u}_j^{n+1} = \bar{u}_j^n + \frac{\Delta t}{\Delta x}\left(F_{j-1/2} - F_{j+1/2}\right) + \Delta t \left\langle g \right\rangle_j^n .$$

It is shown that for steady-state solution $\bar{u}_j^{n+1} = \bar{u}_j^n$.

To obtain high-order schemes we apply Runge–Kutta discretizations in time for Eq. (2) and use WENO approach for high order reconstruction. Instead of constant cell averages $\bar{v}_j$ we compute polynomials $P_j(x)$ satisfying integral equations analogous to (3).

We apply our technique to the shallow water equations and construct well-balanced schemes up to the fourth order of accuracy. The numerical tests show correct treatment of stationary solutions both for still water and moving equilibrium.

Separation of the free surface from a moving body during early stage

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The two-dimensional problems of the liquid flows with separation of the free surface, which are generated by sudden motion of a rigid body or given loads, are considered. Initially the liquid is at rest and the body is in contact with the liquid. The body starts to move either with a non-zero velocity or non-zero acceleration. The generated flow is assumed inviscid and potential. Gravity is included into the model but the surface tension effects are neglected. We need to find the free-surface shape, the flow and pressure distribution during the early stage of the body motion. Four configurations of unsteady problems are studied: (a) impulsive vertical motion of a floating plate; (b) impulsive rotation of a floating plate; (c) impulsive horizontal motion of a half-submerged circular cylinder; (d) motion of a vertical wall with constant acceleration. Small-time asymptotic solutions are derived by the method of matched asymptotic expansions. Second-order outer solutions are obtained analytically and matched with the leading-order inner solutions close to the separation points. The motions of the separation points in problems (b) - (d) are determined by using the condition that the displacement of the free surface is bounded during the early stage. The inner flows close to the separation points for all configurations are self-similar and nonlinear in the leading order with unknown in advance shape of the free surface. The asymptotic solution for the problem (a) was compared with both experimental and numerical results in terms of the free surface shape and the hydrodynamic loads. It was shown that the asymptotic solution can be used even for moderate penetration depths. In the problem (b), the initial position of the separation point was determined by using the Sedov theory of water impact with separation. It was shown that after impact the wetted area starts to grow at infinite velocity. The hydrodynamic pressure is of order of $O(t^{-\frac{1}{3}})$ inside the wetted part of the plate. The inner solution at the separation point predicts the jet flow. The pressure in the contact region for the configuration (c) was found to be below atmospheric pressure if the Froude number is higher than 1.27. The configuration (d) corresponds to the coupled problem of dam-break flow. The motion of the vertical dam wall was determined together with the initial flow and the initial position of the separation point. The obtained solution approaches the classical dam-break solution for small dam mass. The position of the separation point was obtained as a function of the initial acceleration of the dam. Static nonlinear problem of floating ice sheet with unknown positions of the contact points between the ice and the water surface was studied. The ice deflection was due to given external loading acting on the upper surface of the ice sheet. Bending stresses in the ice sheet with and without account for separation were calculated to demonstrate the effect of the free surface separation on the stresses in ice.

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Bernoulli law under minimal smoothness assumptions and applications

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Consider the Euler system

$$\begin{cases} (\mathbf{w}\cdot\nabla)\mathbf{w}+\nabla p &= 0, \\ \operatorname{div}\mathbf{w} &= 0. \end{cases} \tag{1}$$

Let $\Omega\subset\mathbb{R}^2$ be a bounded domain with Lipschitz boundary. Assume that $\mathbf{w}=(w_1,w_2)\in W^{1,2}(\Omega,\mathbb{R}^2)$ and $p\in W^{1,s}(\Omega)$, $s\in[1,2)$, satisfy the Euler equations (1) for almost all $x\in\Omega$ and let $\int_{\Gamma_i}\mathbf{w}\cdot\mathbf{n}\,dS=0$, $i=1,2,\dots,N$, where Γ_i are connected components of the boundary $\partial\Omega$. Then there exists a stream function $\psi\in W^{2,2}(\Omega)$ such that $\nabla\psi=(-w_2,w_1)$ (note that by Sobolev Embedding Theorem ψ is continuous in $\overline{\Omega}$) . Denote by $\Phi=p+\dfrac{|\mathbf{w}|^2}{2}$ the total head pressure corresponding to the solution $(\mathbf{w},p)$.

Theorem 1. *Under above conditions, for any connected set $K\subset\overline{\Omega}$ such that $\psi\big|_K=\mathrm{const}$ the assertion*

$$\exists C=C(K)\quad \Phi(x)=C\quad \textit{for } \mathcal{H}^1\textit{-almost all } x\in K$$

holds.

Theorem 1 was obtained in [1]. Here we denote by $\mathcal{H}^1$ the one-dimensional Hausdorff measure, i.e., $\mathcal{H}^1(F)=\lim\limits_{t\to0+}\mathcal{H}^1_t(F)$, where

$$\mathcal{H}^1_t(F)=\inf\{\sum_{i=1}^{\infty}\operatorname{diam}F_i:\,\operatorname{diam}F_i\le t, F\subset\bigcup_{i=1}^{\infty}F_i\}.$$

Using Theorem 1 we prove the existence of the solutions to steady Navier–Stokes equations for some plane cases (see [2]) and for the spatial case when the flow has an axis of symmetry.

The proof of Theorem 1 relies upon some new analog of Morse-Sard Theorem for Sobolev spaces (see [3]).

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Kolmogorov's theorem for low-dimensional tori of Hamiltonian systems

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The problem of persistence of low-dimensional invariant tori under small perturbation of integrable hamiltonian systems is considered. The existence of one-to-one correspondence between weak hyperbolic invariant tori of a perturbed system and critical points of a smooth function of two real variables is established. It is proved that if the unperturbed Hamiltonian has a saddle point, then a weak-hyperbolic torus persists under an arbitrary analytic perturbation.

Stability issues for two layer flows

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Motivated by the study of internal waves in oceanography, the aim of this talk is to study the propagation of interfacial waves at the interface between two flows. Kelvin-Helmholtz instabilities are known to occur in such situations, but we will describe various mechanisms that prevent their formation and allow the waves to exist. Some applications will be given.

On the super-critical wave equation

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In this talk, we will discuss some results on the Cauchy problem for the defocusing nonlinear wave equation in $\mathbb{R}^3$:

$$(E_p): \quad (\partial_t^2 - \triangle_x)u + u^p = 0$$

with data in the energy space $H^1 \times L^2$, and p odd.

This problem is globally well posed for $p \leq 5$, and admits weak solutions for any odd values of the exponent p. The problem of existence and uniqueness of strong solutions for $p \geq 7$ is still open. We will recall some results on the instabilities of solutions in the super-critical case $p \geq 7$, (including semi-classical versions of (E_p)), and also results on the local existence of strong solutions for almost all data.

Internal solitary waves: direct and inverse problem

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We consider the problem on large amplitude internal solitary waves propagating in a weakly stratified fluid under gravity. It is well known that steady 2D Euler equations of non-homogeneous fluid can be reduced to single quasi-linear elliptic equation, this is the Dubreil – Jacotin — Long equation for a stream function. By that, extreme shapes of solitary waves have been studied [1, 2] in the framework of the DJL equation. Analytic conditions which guarantee bifurcation of limit forms of internal waves were obtained. These conditions are formulated in the terms of fine-scale density profile over background linear- or exponential stratification. In addition, amplitude bounds for solitary waves, imposed by the conservation of mass, momentum and energy, were found [3] on the basis of conjugate flow theory. In this paper, we discuss some properties of integral equations coupling the fluid density coefficient with the dispersion function.

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On existence and uniqueness of solutions in several boundary-value problems for the Euler equations

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We consider several boundary-value problems for the Euler equations describing flows of an ideal incompressible fluid in a bounded domain: the problem NP with non-penetration condition at the boundary as well as so called through-flow problems describing flows of the fluid through the domain, with different types of boundary conditions. Namely, three types of through-flow problems are considered. In the first problem (denoted as TF-I) the vorticity is prescribed at the entrance (i. e., at the part of the boundary where the fluid enters the domain). More exactly, in the 3D flows we should prescribe only the tangent components of the vorticity, and, in any dimensions, the normal component of the velocity is prescribed at the whole boundary (except the exit, in some versions). In the problems TF-II and TF-III the whole velocity vector is prescribed at the entrance and the pressure or the normal component of the velocity, respectively, are specified at the exit. We are interested in global (in time and input data) existence and uniqueness theorems for these problems formulated in classes of solutions as wide as it is possible. Such interest if stimulated by well-known global existence problem for 3D Euler equations, which is strongly connected with the study of irregular solutions. Thus, we have to study nonsmooth solutions of the Euler equations and prove their existence and uniqueness.

We present two main results. The first result consists in uniqueness of solutions of the problems NP, TF-II and TF-III (for any dimensions of the flow) in the classes with non-bounded vorticity. These classes are presented using the Orlicz classes and seem to be easy to verify in applications since they are formulated in a rather clear form as against well-known results. The second result consists in global existence theorem for the 2D problem TF-I in the classes of solutions with non-bounded vorticity that belongs to the Lebesgue spaces L_p with $p > 4/3$; it seems to be interesting as soon as well-known results concerning nonsmooth solutions of 2D Euler system do not cover through-flow problems.

Our methods discover curious relations of the named problems with the theory of integral transforms and representations of functions. Some details can be found in [1, 2].

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Factorization for non-symmetric operators and exponential H-theorem

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We present a factorization method for estimating resolvents of non-symmetric operators in Banach or Hilbert spaces in terms of estimates in another (typically smaller) "reference" space. This applies to a class of operators writing as a "regularizing" part (in a broad sense) plus a dissipative part. Then in the Hilbert case we combine this factorization approach with an abstract Plancherel identity on the resolvent into a method for enlarging the functional space of decay estimates on semigroups. In the Banach case, we prove the same result however with some loss on the norm. We then apply this functional analysis approach to several PDEs: the Fokker-Planck and kinetic Fokker-Planck equations, the linear scattering Boltzmann equation in the torus, and, most importantly the linearized Boltzmann equation in the torus (at the price of extra specific work in the latter case). In addition to the abstract method in itself, the main outcome of the paper is indeed the first proof of exponential decay towards global equilibrium (e.g. in terms of the relative entropy) for the full nonlinear Boltzmann equation for hard spheres, conditionally to some smoothness and (polynomial) moment estimates. This improves on the result by Desvillettes and Villani where the rate was "almost exponential", that is polynomial with exponent as high as wanted, and solves a long-standing conjecture about the rate of decay in the H-theorem for the nonlinear Boltzmann equation.

Dynamics of the flows through a finite channel with Yudovich's boundary conditions

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This communication concerns the open flows such as flows through a finite channel or a pipe with inflow of fluid through one end and outflow through the other end. Then the fluid fluxes supply to and withdraw from the flow both the energy and vorticity. The supply/withdrawal competition depends on the boundary conditions that can be formulated in different ways. We restrict ourself within planar channel flows subject to Yudovich's boundary conditions (YBC,1963): the normal velocity is specified everywhere on the boundary, and, in addition, the vorticity is specified on the flow inlet.

We give a constructive description to the wide class of YBC's for which the flow dynamics is essentially dissipative. Very roughly, the dissipative bc's prescribe a monotonic dependence between the vorticity and the stream function on the inlet. Each flow subject to the dissipative bc's admits the functional of Arnold (1966) which is not the constant of motion but represents the decreasing Liapunov function. Its derivative concentrates itself on the flow outlet (in sharp contrast with the case of the viscous dissipation).

For the dissipative bc's, we discover two qualitatively different patterns of the flow behaviour: some flows are able to wash out the excessive vorticity while others are able to trap it. The flows of former kind evolve to the non-separated steady flow which is determined by the bc's completely. The flows of the latter kind evolve to separated flows which may represent rather complex vortical configurations. These configurations depend on the initial states of the flows essentially. The non-dissipative bc's give rise to one more scenario. This is the onset of self-oscillations.

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On discontinuity solutions of shallow water equations in channel with cross section jumping

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The equations of the first approximation of shallow water theory [1] are widely used [2] to model the propagation of hydraulic bores generated by total or partial dam breaks. The theory describes such bores as steady discontinuous solutions, which we, following [3] will name shocks. However, the classical system of the basic shallow water conservation laws, while correctly describing the parameters of hydraulic bores propagating in a regular channel, does not allow describe the water flows on cross section jumping. It is caused by that the equation of full momentum is the exact conservation law only in the case of regular channel. Therefore it cannot be used for obtaining the the Rankine-Hugoniot conditions on the discontinuous arising on cross section jumping. From this it follows that if on such jumping come two characteristics than it is necessary introduction of an additional condition on the discontinuous.

In present paper, following [4, 5], we will obtain such additional condition from local momentum conservation law which conserve the divergent form in the case of nontrapezoidal channel. Consequence of it is conservation on such discontinuous of full energy of a running stream. As a concrete example it is considered a dam break problem on cross section jumping. Unequivocal resolvability of this problem within the limits of the solutions containing stationary shocks and centered depression waves is investigated.

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On the propagation of oceanic waves driven by a macroscopic current

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We study oceanic waves in a shallow water ow subject to strong wind forcing and rotation, and linearized around a inhomogeneous (non zonal) stationary prole. This extends an eralier the study, where the prole was assumed to be zonal only and where explicit calculations were made possible due to the 1D setting. Here the diagonalization of the system, which allows to identify Rossby and Poincare waves, is proved by an abstract semi-classical approach based on normal forms. The dispersion of Poincare waves is also obtained by a more abstract and more robust method using Mourre estimates.

The coalescence and resonant breakup of pulsating gas bubbles in fluid

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The convergence and coalescence of pulsating gas bubble in fluid can be explained by the Bjerknes force. However, the experiments show that the coalescence of bubbles is not always observed. The condition of coalescence can be obtained by taking into consideration the forces of viscosity. The case of two pulsating bubbles with equal radii was considered. A period-mean force between pulsating bubbles and the Lagrange function were obtained with the help of the method of reflections.

The condition of coalescence was obtained $A\rho a\omega/\mu > 6$, where a is the radii of bubbles, ω is frequency of pulsation μ is the viscosity coefficient of fluid, A is amplitude of bubble pulsation (Petrov 2010). This condition is in accordance with the experiment and the previous works on this topic (Boshenyatov 2009).

One of possible mechanisms of bubble breakup is one due to shape instability. The resonance of radial and arbitrary deformational oscillation mode frequencies 2:1 was examined using the Zhuravlev's method of invariant normalization of Hamiltonian systems and the complete analytical solution was obtained. It has been shown the problem is fully analogical to that of the swinging spring (Petrov 2006). It has been shown that in order for arbitrary oscillation mode described by a high Legendre polynomial number n to be in resonance 2:1 with the radial bubble oscillations the bubble radii should be proportional to 3n. The exact expression for energy transfer period was obtained. The maximal magnitude of deformational mode due to energy transfer from radial mode has been shown to grow with the growth of n linearly. For example the relation of magnitudes in case of n=7 is about 20.

The resonant energy transfer and huge growth of deformational mode magnitude on big n could be the explanation of bubble breakup due to shape instability (Vanovskii, Petrov 2011). *The work was supported by RFFI 11-01-00535*

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On the stationary Navier-Stokes system with non-homogeneous boundary data

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First, the nonhomogeneous boundary value problem for the Navier–Stokes equations

$$\begin{cases} -\nu\Delta\mathbf{u} + \big(\mathbf{u}\cdot\nabla\big)\mathbf{u} + \nabla p = 0 & \text{in } \Omega, \\ \operatorname{div}\mathbf{u} = 0 & \text{in } \Omega, \\ \mathbf{u} = \mathbf{a} & \text{on } \partial\Omega \end{cases} \tag{1}$$

is studied in a bounded multiply connected domain $\Omega \subset \mathbb{R}^n$ with the boundary $\partial\Omega$, consisting of N disjoint components Γ_j.

Starting from the famous J. Leray's paper published in 1933, problem (1) was the subject of many papers. The continuity equation in (1) implies the necessary solvability condition

$$\int_{\partial\Omega} \mathbf{a}\cdot\mathbf{n}\,dS = \sum_{j=1}^{N}\int_{\Gamma_j} \mathbf{a}\cdot\mathbf{n}\,dS = 0,$$

where $\mathbf{n}$ is a unit vector of the outward (with respect to Ω) normal to $\partial\Omega$. However, for a long time the existence of a weak solution $\mathbf{u} \in W^{1,2}(\Omega)$

to problem (1) was proved only under the stronger condition

$$\mathcal{F}_j = \int_{\Gamma_j} \mathbf{a} \cdot \mathbf{n} \, dS = 0, \qquad j = 1, 2, \dots, N.$$

Problem (1) is studied in a two–dimensional bounded multiply connected domain $\Omega = \Omega_1 \setminus \Omega_2$, $\overline{\Omega}_2 \subset \Omega_1$, with Lipschitz boundary. It is proved that it has a solution, if the flux $\mathcal{F} = \int_{\partial\Omega_2} \mathbf{a} \cdot \mathbf{n} \, dS$ of the boundary value through $\partial\Omega_2$ is nonnegative (outflow condition). This result was obtained jointly with M. Korobkov and R. Russo.

Second, problem (1) is studied in domains with noncompact boundaries. The existence of at least one solution is proved without restrictions on fluxes of the boundary value over the "outer" connected components of the boundary and without any symmetry assumption on the domain. The fluxes over "inner holes" are assumed to be sufficiently small. The obtained solutions could have either finite or infinite Dirichlet integral over the outlets to infinity depending on the geometrical properties of these outlets. These results are obtained jointly with PhD student K. Kaukakyte.

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Self-improving bounds for Navier-Stokes and applications to Serrin's endpoint criterion

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Solutions which are time-bounded in L^3 up to time T can be continued past this time, by a landmark result of Escauriaza-Seregin-Sverak, extending the well-known Serrin's criterion. On the other hand, the local Cauchy theory holds up to solutions in BMO^{-1}; we aim at describing how one can obtain regularity results assuming a priori bounds in negative regularity Besov spaces which are "in between" L^3 and BMO^{-1}.

For this we rely crucially on improving bounds from negative to positive regularity.

Stationary viscous flows in domains with multiply connected boundary (the Jean Leray problem)

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We consider stationary Navier-Stokes equations describing motion of an incompressible viscous liquid in a bounded domain with a boundary, which has several connected components. The velocity vector is given on the domain boundary and liquid fluxes differ from zero on its components. We provide a survey of previous results, which deal with partial versions of the problem based on symmetry flow properties or smallness assumptions of fluxes. We construct an a priori estimate of the Dirichlet integral for velocity vector in the case, when the flow has a symmetry plane perpendicular to this axis; moreover this plane intersects each component of the boundary. The last condition can be omitted if values of stream function and vorticity are given on the domain boundary. Having available a priori estimate, we prove the existence theorem for axially symmetric problem in a domain with a multiply connected boundary. Besides, a special case of the plane problem is considered, where the domain boundary consists of a Jordanian curve and a point inside domain. In this case, the Dirichlet integral of velocity is infinite.

Roll waves in shear shallow water flows on an inclined plane

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Roll waves are a periodic system of smooth profiles separated by hydraulic jumps. The existence of roll waves has been proved by Dressler (1949) by using classical shallow water equations on an inclined plane with friction. The stability of Dressler's roll waves has been studied by Lyapidevsky and Teshukov (2000), Noble (2006) and Johnson, Zumbrun and Noble (2010).

However, the experimental wave profiles (Brock, 1967) are completely different from Dressler's solution (see Figure 1). We adapted the model proposed by Teshukov (2007) for description of shear shallow water flows. This model is analogous to the model of turbulent flows of compressible fluids supplemented by "production-dissipation" terms. The roll waves solutions have been obtained. A very good agreement of theoretical and experimental profiles has been observed.

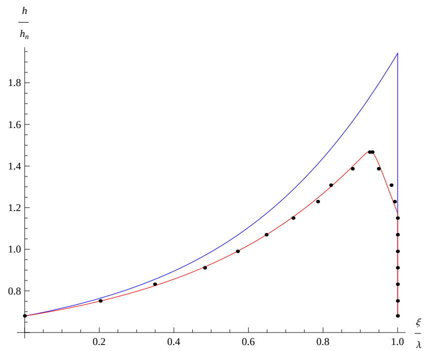

Figure 1: Comparison between Brock's experiments (dots) and theory in the plane "wave length-wave profile". The red curve is an exact solution to our model, the blue curve is the Dressler solution.

Uniform regularity and inviscid limit for the free surface Navier-Stokes equation

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We consider the incompressible Navier-Stokes equation in a fluid domain limited by a free surface. We shall prove that under a Rayleigh condition, we can get an existence theory which provides a life time which is uniform when the viscosity goes to zero. This allows to justify the inviscid limit by strong compactness arguments.

Homogenization of time-harmonic Maxwell equations and the frequency dispersion effect

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We perform homogenization of the time-harmonic Maxwell equations in order to determine the effective dielectric permittivity ε^h and effective electric conductivity σ^h. We prove that ε^h and σ^h depend on the pulsation ω; this phenomenon is known as the frequency dispersion effect. Moreover, the macroscopic Maxwell equations also depend on ω; they are different for small and large values of ω.

Recent advances in the three-dimensional Wagner problem

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The so-called linearized three-dimensional Wagner problem for hydrodynamic impact is formulated in Potential theory as a mixed boundary value problem posed in a half space $z < 0$. The plane $z = 0$ is the boundary of the linearized fluid domain. It is described with coordinates (x, y). It is broken down into a free surface FS supporting a homogeneous Dirichlet condition and the impacting wetted surface D supporting a known Neumann condition; the latter is expressed in terms of the shape function $f(x, y)$ and the penetration depth h. The main difficulty originates from the representation of the contact line $\Gamma = FS \cap D$ which is a part of the solution. The boundary value problem can be solved via variational inequalities as done by [1]. More recently, by using drastic (but smart) assumptions, [2] proposed algorithms for some general shapes with good agreement both with experimental and numerical (CFD) results. In practice the boundary value problem can reduce to an integral equation for planar Laplacian $\Delta_2\Phi = \Phi_{,x^2} + \Phi_{,y^2}$ of the displacement potential Φ as described in [3] and [4]. It can be shown that $\Delta_2\Phi$ has necessarily a square root singularity along Γ; provided that Γ is smooth enough (see [5]). It is also shown that there is a strong connection between the regularity of the shape function $f(x, y)$ and the regularity of integrand $\Delta_2\Phi$. In particular Galin's theorem gives (see [8]) quasi-analytical solutions of the integral equation provided that contact line $\Gamma(t)$ is elliptic. In this direction new solutions can be found "in the footsteps" of [6] and [7]. The present work is not precisely a step ahead in the domain of three-dimensional Wagner problem but it is rather a state of the art in order to underline the main encountered difficulties and probably the last ones to overcome.

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Nonlinear equations of the first kind with homogenous operators

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This paper presents the theoretical studies of behavioral modeling and control of nonlinear dynamical processes. The impulsive control problem is formulated in terms of generalized solutions of one class of polynomial integral equations of the first kind

$$\sum_{n=1}^{N}\sum_{j=1}^{\infty}\prod_{i=1}^{n}\int_{0}^{t} K_{nij}(t,s)\,x(s)\,ds = f(t).$$

Such equations appear in the nonlinear systems theory to control the behavior of nonlinear dynamical processes based on the Volterra models [1]. The existence theorem is formulated for this class of equations and the method of construction of generalized solutions as sum of the Dirac delta functions and regular continuos functions is proposed. It is demonstrated that the number of solutions is equal to the number of roots of the certain polynomial. Some remarks on construction of classic continuos solutions to control problem are presented. For more details regarding the continuos solution of polynomial integral equations readers may refer to [2, 3].

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Some equations with Fredholm operator in main part: existence theorems, bifurcation and applications

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Creation and investigation of parameter dependent functional equations in mathematical models with Fredholm operator on the main part has various applications in many areas of mathematical modeling. Contemporary branching (bifurcation) theory is one of the most important aspects in state of the art applied mathematics. The goals of this theory are the qualitative theory of dynamical systems, analytical and numerical computation of their solutions under absence of conditions guaranteeing the uniqueness of the solution. Applications sphere of bifurcation theory and Lyapunov-Schmidt method is permanent extending. Besides their traditional applications in elasticity theory and hydrodynamics bifurcation theory methods turn out to be successful in the investigation of specific nonlinear problems of phase transitions and plasma physics, mathematical biology, filtration theory, non-Newtonian fluids movement theory. In recent decades the Lyapunov-Schmidt method has been applied in combination with representation and group analysis theories, finite-dimensional topological and variational methods, perturbation methods as well as the regularization theory (see the monographs [1, 2]). Such combined methods approaches have given the possibility to prove the most general existence theorems of bifurcating solutions, to make their algorithmic and qualitative analysis, to develop asymptotical and iterative methods for differential-operator equations. This report presents some our results in the above-mentioned areas. They contain the corresponding general theories of classical and generalized solutions of operator and differential-operator equations in Banach spaces with Fredholm operators in main part illustrating by applications both to boundary value problems for partial differential equations and to integral and integro-differential equations.

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Numerical study of convective vortex (dependencies of the integral characteristics on the boundary conditions)

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G.P. Bogatyrev and his colleagues studied the laboratory model of typhoon-like vortex in Perm since 1990 [1, 2]. This research was focused on formation and evolution of an intensive convective vortex in rotating cylindrical vessel with local heating from a bottom. According to the experiments, vortex velocity exceeded a solid-state rotation in about 10 times. After the laboratory experiments several articles devoted to numerical modeling of similar phenomenon were published [3, 4]. In this work we also use resources of numerical modeling to study dependencies which were not investigated earlier. Our research focuses on integral characteristics, conditions of intensive vortex formation and axial flow restructuring with changing of flow direction. The axysimmetric problem, described by the heat convection equations (Boussinesq approximation), was solved using finite differences in the variables of two-field method. Geometrical parameters corresponded to the laboratory experiments [1]. Except geometrical parameters problem had three dimensionless parameters — Grashof, Prandtl and Reynolds numbers. Flow pictures for several flow types were obtained. Boundaries of intensive vortex formation and axial flow restructuring were determined earlier [5]. In this work we investigate integral characteristics (kinetic energy, Nusselt number, integral angular momentum, global super-rotation [2]) dependencies on the boundary conditions (heat area, aspect ratio, side boundary type). Comparison with laboratory experiments results and tests on different grids were performed.

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Existence and stability of relativistic plasma-vacuum interfaces

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The equations of relativistic magnetohydrodynamics in the Minkowski spacetime (t,x) are written as a system of conservation laws and then as the symmetric hyperbolic system

$$A_0(U)\partial_t U + A_1(U)\partial_1 U + A_2(U)\partial_2 U + A_3(U)\partial_3 U = 0 \qquad (1)$$

for the vector $U = (p, u, H, S)$ of "primitive" variables, where p is the pressure, $u = v\Gamma$, $\Gamma = (1-|v|^2)^{-1/2}$, v is the 3-velocity, H is the magnetic field 3-vector, and S is the entropy. A concrete form of symmetric matrices A_α was recently found in [1]. The vacuum Maxwell equations $\partial_t\mathcal{H} + \nabla\times E = 0$, $\partial_t E - \nabla\times\mathcal{H} = 0$ for the electromagnetic field $V = (E,\mathcal{H})$ also form a symmetric system in the form of (1) with $A_0 = I$ and constant matrices A_j. Moreover, we have the divergence constraints $\operatorname{div} H = 0$, $\operatorname{div} E = 0$ and $\operatorname{div}\mathcal{H} = 0$ on the initial data $(U,V)|_{t=0} = (U_0, V_0)$.

Let $\Omega^\pm(t) = \left\{x^1 \gtrless \varphi(t,x^2,x^3)\right\}$ be the domains occupied by the plasma and the vacuum respectively. Then, on the interface $\Sigma(t) = \{x^1 = \varphi(t,x^2,x^3)\}$ we have the conditions

$$\begin{gathered}\partial_t\varphi = v_N, \quad q = (|\mathcal{H}|^2 - |E|^2)/2,\\ E_2 = \mathcal{H}_3\partial_t\varphi - E_1\partial_2\varphi, \quad E_3 = -\mathcal{H}_2\partial_t\varphi - E_1\partial_3\varphi,\end{gathered} \qquad (2)$$

where $q = p + |H|^2/(2\Gamma^2) + (v,H)^2$ is the total pressure and v_N is the normal component of the velocity. Moreover, the conditions $H_N|_\Sigma = 0$ and $\mathcal{H}_N|_\Sigma = 0$ are restrictions on the initial data, and we assume that the plasma density $\rho|_\Sigma > 0$.

Our final goal is to find conditions on the initial data (U_0, V_0, φ_0) providing the local-in-time existence and uniqueness of a smooth solution (U, V, φ) of the free boundary problem for system (1) in $\Omega^+(t)$ and the

Maxwell equations in $\Omega^-(t)$ with the boundary conditions (2) on $\Sigma(t)$. Following [2, 3], we use the reduction to a fixed domain, the passage to Alinhac's "good unknown" (see [2]), and a suitable Nash-Moser-type iteration scheme. Since the interface is a characteristic surface, as in [2], the functional setting is provided by the anisotropic weighted Sobolev spaces H^m_*. The crucial point is finding a sufficient stability condition for a planar plasma-vacuum interface. This condition was found in [4] by a so-called *secondary symmetrization* of the vacuum Maxwell equations.

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